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Regional business cycle synchronization through expectations

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Abstract

This paper provides an example in which regional business cycles may synchronize via producers' expectations, even though there is no interregional trade, by means of a system of globally coupled, noninvertible maps. We concentrate on the dependence of the dynamics on a parameter η which denotes the inverse of price elasticity of demand. Simulation results show that several phases (the short transient, the complete asynchronous, the long transient and the intermediate transient) appear one after another as η increases. In the long transient phase, the intermittent clustering process with a long chaotic transient appears repeatedly.

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1. Introduction

A national economy consists of diverse regional sub-economies, and national business cycles are an admixture of regional cycles. One of the stylized facts about regional cycles is that fluctuations in different regions of a national economy are inclined to synchronize with each other [1–3].

One possible explanation of this synchronization may be that it occurs due to common exogenous shocks such as national fiscal and/or monetary policies, sudden changes in world commodity prices, fads among consumers, and so forth. However, several studies find that common shocks do not seem to be the cause of such synchronization [4,5].

Another possible explanation is that the synchronization is caused by trade linkages between different regions of the economy. A nonlinear mode-locking model is proposed as a mechanism[1] [7], taking into consideration the empirical evidence which suggests that the aggregation process of regional cycles might be

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[1]There are many studies applying a mode-locking mechanism to some other aspects of business cycles, which are surveyed briefly by Süssmuth [6].

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more adequately described by a nonlinear process than by a linear one [8]. Mode-locking is an inherently nonlinear linkage phenomenon; cycles of different elements are synchronized, i.e., attain 'mode-lock', when the strength of the linkage between oscillating elements reaches a certain threshold. A scenario in which cycles of different regions may synchronize due to trade linkages among the regions is probable and realistic, but rather straightforward and obvious.

In the present paper, we propose another possible explanation, i.e., an explanation that expectations based on the average level of the economy may induce regional business synchronization, by means of a system of globally coupled, noninvertible maps [9]. A prototype of the globally coupled map (GCM) is represented by

$$x_{t+1}(i) = (1-\varepsilon)f(x_t(i)) + \frac{\varepsilon}{N}\sum_{j=1}^{N} f(x_t(j)), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_t(i)$ is the variable of the ith element at discrete time step t, and $f(x)$ represents the endogenous dynamics of each element. For the endogenous dynamics, a noninvertible map which exhibits chaotic behavior is utilized. Since the second term on the right-hand side of (1) represents the global interaction of each element through the mean field, in a system of GCM there are many dynamical elements interacting all-to-all. Two opposite effects coexist: all-to-all coupling is inclined to synchronize elements of the model whereas chaotic instability in each element tends to desynchronize them. Depending upon the balance between these two effects, the GCM model exhibits a rich variety of complex phenomena [9].

2. Model

The economy is divided into N regions, each of which has a separate market. For simplicity, let us suppose that there is only one producer in each region, so that there exist N producers in total in the economy. The producer in each region produces homogeneous goods and supplies them to the market he operates in. Consumers purchase goods from the market they belong to. Since the main focus of the present paper is to show how regional business cycles synchronize through expectations, we perform a kind of thought experiment, in order to segregate expectation effects from trade-linkage effects, and assume that there is no interregional trade. Instead the regional markets are connected with each other in the following way: The government announces the average price and the average output of the whole economy at each time period and the producer decides his production plan for the next period based on such average information and other factors. In what follows, a system of GCM is derived based on a cobweb-type model[2] with adaptive adjustment [10].

At period t, ith producer expects that actual prices $p_t(i)$ in his market will be adjusted adaptively at period $t+1$ toward the average level $\bar{p}_t = (1/N)\sum_{j=1}^{N} p_t(j)$ announced by the government, so that

$$p^e_{t+1}(i) = (1-\varepsilon)p_t(i) + \frac{\varepsilon}{N}\sum_{j=1}^{N} p_t(j), \tag{2}$$

where the superscript e denotes expectation and $\varepsilon \in (0,1)$ is an expectation adjustment coefficient common to all producers.[3]

Under this price expectation, the expected profit of producer at period $t+1$ is given by $p^e_{t+1}(i)x_{t+1}(t) - C(x_{t+1}(i))$, where $C(x) = x^2/2$ is a cost function. Thus the output which maximizes the expected profit is given by

$$\tilde{x}_{t+1}(i) = p^e_{t+1}(i). \tag{3}$$

In order to hedge against the risk of failure in expectations, producer produces the amount which is a weighted average of the present period's output $x_t(i)$, the average output of the whole economy announced by the government $\bar{x}_t = (1/N)\sum_{j=1}^{N} x_t(j)$, and the output which maximizes the expected-profit $\tilde{x}_{t+1}(i)$. The basic idea

[2]The cobweb model is an economic model which explains why prices and outputs in certain markets are subject to periodic fluctuation. It shows an adjustment process that, on a supply–demand diagram, spirals toward an equilibrium like in a cobweb.

[3]As will be described below, actual prices $p_t(i)$ are determined in the market so as to equilibrate supply and demand.

of the weighted average is as follows: (i) producer dislikes the deviation from the average level, so that he adjusts the output toward the average level with the adjustment coefficient of ε which is equal to the coefficient of price-expectation adjustment discussed above; the resulting amount is $(1-\varepsilon)x_t(i)+\varepsilon\bar{x}_t := \hat{x}_t(i)$, and (ii) he calculates the profit-maximizing amount $\tilde{x}_{t+1}(i)$ as a target of adjustment and adjusts $\hat{x}_t(i)$ toward it with the coefficient of $\phi \in (0,1)$, which is common to all producers. The resulting formula is given by

$$x_{t+1}(i) = (1-\phi)(1-\varepsilon)x_t(i) + (1-\phi)\varepsilon\bar{x}_t + \phi\tilde{x}_{t+1}(i). \tag{4}$$

Evidently, the sum of the above three coefficients is unity, which implies that (4) is, in fact, a weighted average of $x_t(i)$, $\bar{x}_t$, and $\tilde{x}_{t+1}(i)$. Since we assume that there is only one producer in each region, total supply of each regional market is given by (4).

Let us assume that the demand of each region is identical and described by the same monotonic inverse demand function

$$p_t(i) = \frac{1}{(y_t(i))^\eta}, \tag{5}$$

where $y_t(i)$ is the demand of ith region at period t and $\eta > 0$ is the inverse of the price elasticity of demand.[4] Finally, we assume that, at each period, prices are determined in the market so as to equilibrate supply and demand;

$$x_t(i) = y_t(i). \tag{6}$$

Substituting (2), (3), (5), and (6) into (4) yields a system of GCM (1), where

$$f(x_t(i)) = (1-\phi)x_t(i) + \frac{\phi}{(x_t(i))^\eta}. \tag{7}$$

Our model differs from the standard GCM model in that (7) is utilized in the former, while the logistic map is used in the latter. The behavior of (7) is studied well [10] and known to exhibit chaotic behavior depending upon the set of parameters; the larger ϕ and η are, the more likely it behaves chaotically via period-doubling bifurcations.[5]

3. Simulations

Since the GCM not only has very complex structure in the parameter space but also depends on initial conditions, three parameters are fixed as follows in all simulations; $\varepsilon = 0.1$, $\phi = 0.8$, $N = 100$. For initial conditions we start with $x_0(i) = x^* + \xi$, where $x^* = 1.0$ is the unique fixed point of the map (7) and $\xi \in [-0.5, 0.5]$ is a uniformly distributed random number. In the present paper, we investigate η dependence of the dynamics.

Typical time series are depicted in Fig. 1 for different values of η. For smaller $\eta \in [1.0, 2.8]$, starting from the above initial conditions, all regions are divided into a small number of clusters (typically 1 or 2, depending on initial conditions) after a short transient time. In other words, if the price elasticity of demand is relatively large, synchronous clusters are formed soon and these clusters oscillate periodically (note that, in this parameter region, a single map dynamics is periodic). For $\eta \in [3.1, 3.6]$, each $x(i)$ is completely asynchronous, i.e., $x(i) \neq x(j)$ for $i \neq j$. That is to say, if the demand is more inelastic, there would not come about any synchronous clusters. We check that $|x(i)-x(j)| > \delta = 10^{-8}$ for all $t \leqslant \mathcal{O}(10^8)$ (see Fig. 1(b) where time is plotted on the logarithmic scale). For $\eta > 6.2$, synchronous clusters are formed (the number of clusters is typically 1 or 2) after an intermediate transient time. To put it differently, if the demand is fairly inelastic, synchronizations occur again only after an intermediate transient. After a transient, the dynamics of one or two clustering regions is chaotic or periodic depending on both η and initial conditions. A long transient

[4]The price elasticity of demand measures the relationship between changes in the prices of a good and changes in quantity demanded of that good. It is defined as $\sigma_{p,y} := |(\mathrm{d}y/y)/(\mathrm{d}p/p)|$ and, with respect to (5), is calculated as follows:

$$\sigma_{p,y} = \left|\frac{p}{y}\cdot\frac{\mathrm{d}y}{\mathrm{d}p}\right| = \left|\frac{1}{y^{\eta+1}}\cdot\frac{1}{-\eta y^{-\eta-1}}\right| = \frac{1}{\eta}.$$

[5]An extension of the model (7) to two types of producers, cautious adapters and naive optimizers, is carried out [11].

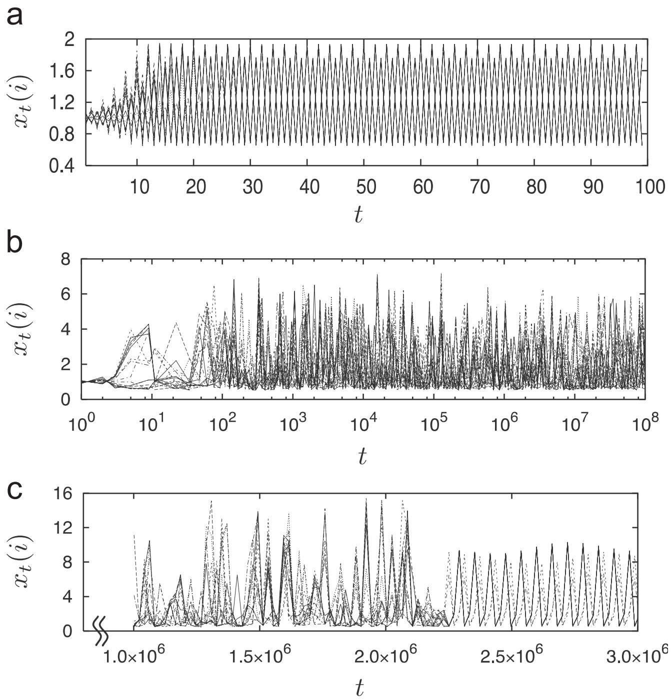

Fig. 1. Typical time series of $x(i)$. Twenty elements ($x(i), i = 1, \ldots, 20$) are plotted (using different line types) for clarity. (a) $\eta = 2.0$: All regions are divided into two clusters after a short transient time. (b) $\eta = 3.4$: Complete asynchronous state: where all $x(i)$'s have different values. The logarithmic time scale is used to see the 'completeness' of asynchronous state. (c) $\eta = 4.2$: All regions are divided into three clusters after a long transient time.

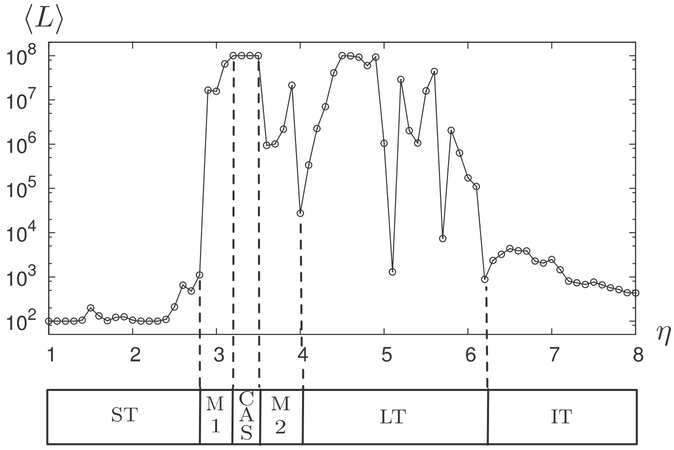

Fig. 2. Schematic phase diagram and lifetime of asynchronous state are shown. *ST*: Short transient phase with one or two clusters. *M*1: Mixture of ST and CAS depending on initial conditions. *CAS*: Complete asynchronous phase with virtually 100 clusters. *M*2: Mixture of CAS and LT depending on initial conditions. *LT*: Long transient phase with a small number of clusters (from 1 to about 10). *IT*: Intermediate transient phase with one or two clusters (almost 2).

dynamics is observed in $\eta \in [3.9, 6.2]$ where several synchronous clusters are formed after a long time evolution (see Fig. 1(c)) [12].

In order to investigate the transient dynamics, we calculate the lifetime L of the asynchronous state, i.e., how long the asynchronous state lasts starting from randomly selected initial conditions. The asynchronous state is defined as the state in which the number of clusters is larger than $n_c = 5$ (L does not significantly

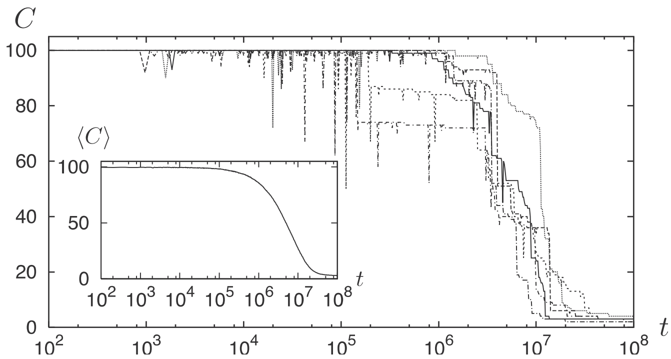

Fig. 3. The time evolution of the number of clusters C for $\eta = 5.5$ starting from six different initial conditions (using six different line types) is shown. The evolution of C has plateaus and eventually falls. The elements finally converge to a periodic state consisting of one or two clusters. The inset shows the evolution of C averaged over 1000 different initial conditions. Long-time transient dynamics is clearly observed.

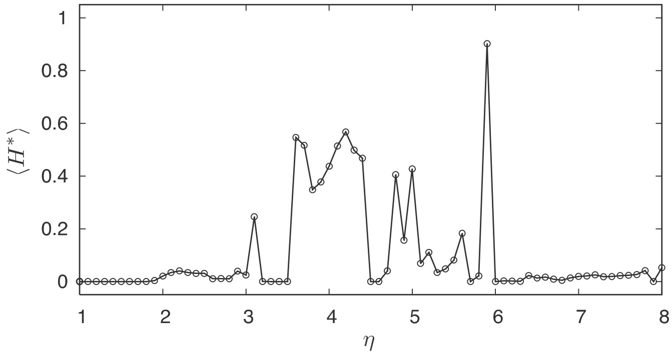

Fig. 4. The normalized nonuniformity of cluster size $\langle H^* \rangle$ averaged over 300 different initial conditions. In ST and IT phases, the number of clusters is 1 or 2, and the clusters are of an almost uniform size. In CAS phase, virtually 100 clusters are a uniform size of 1. Nonuniform cluster size appears in M1, M2, and LT phases.

depend upon n_c). The η dependence of L averaged over 100 different initial conditions and the corresponding schematic phase diagram are shown in Fig. 2. Several phases are clearly observed: the short transient (ST) phase with one or two clusters, the complete asynchronous (CAS) phase with almost all 100 clusters, the long transient (LT) phase with a small number of clusters (from 1 to about 10), and the intermediate transient (IT) phase with one or two clusters (almost 2).

In LT phase, an interesting clustering process is observed. Hereafter we use the number of clusters to characterize this synchronizing process. The number of clusters C is counted as follows: if $|x(i) - x(j)| < \delta = 10^{-8}$, then $x(i)$ and $x(j)$ are regarded as belonging to the same cluster (the results do not significantly depend on the parameter δ). The clusters are eventually formed after long chaotic transients and these clusters are 'quasi-stable' in the sense that the number of clusters persists over a long time. However, these clusters eventually merge again and achieve a synchronous state consisting of a smaller number of clusters. The time series of C in LT phase is shown in Fig. 3 which indicates that the evolution of C consists of long plateaus and periods of sudden drop. This means that the merging of clusters occurs intermittently. The inset of Fig. 3 shows the cluster number averaged over 1000 different initial conditions. Asynchronous state lasts over a period 10^6 starting from 100 regional elements with different initial state values. After a long transient chaotic dynamics, several elements suddenly merge and form synchronous clusters.

In order to characterize attractors (note that there are many attractors even with a fixed set of parameters in this system), we introduce the Herfindahl index, $H = \sum_{i=1}^{C} (n_i/N)^2$ where C is the number of clusters and n_i is the number of elements in a cluster. The index H measures the nonuniformity of cluster size, and was first employed in economics for indicating the concentration ratio of industries [13,14].[6] Here we use a normalized Herfindahl index defined as $H^* = (H - 1/C)/(1 - 1/C)$.[7] If $H^* = 0$, it implies that all clusters are of a uniform size and, given C, H^* increases as the nonuniformity increases. The η dependence of H^* averaged over 300 initial conditions is plotted in Fig. 4. In ST and IT phases, the number of clusters is 1 or 2, and the clusters are of an almost uniform size. In CAS phase, the number of clusters is virtually 100 with a uniform size of 1, so that $H^* \approx 0$. Nonuniform cluster size appears in M1, M2, and LT phases. Spotty economic boom with respect to regions may be observed when regions are divided into multiple clusters.

4. Concluding remarks

The present paper has proposed a new hypothesis that expectations based on the average level of the economy might induce regional business synchronization. The hypothesis uses a GCM model derived from the microeconomic settings of boundedly rational producers. Here we concentrate on the dependence of dynamics on η, which is the inverse of price elasticity of demand. Simulation results show that there appear various phases depending upon η; these are: the perfect synchronization (coherent) phase, CAS phase, and the multiple cluster phase in between. Furthermore, the lifetime of the asynchronous, transient state crucially depends also upon η. ST, CAS, LT, and IT phases appear one after another as η increases. In LT phase, the intermittent clustering process with a long chaotic transient appears repeatedly.

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[6] It is defined as the sum of the squares of the market shares of the relevant individual firms in economics. In physics, the same index was first introduced in the spin glass [15] and is applied to complex chaotic dynamics [16].

[7] The index H ranges from $1/C$ to 1 while the index H^* ranges from 0 to 1. When $C = 1$, it is assumed that $H^* = 0$ to avoid division by zero.