# Speculative Dynamics in a Heterogeneous-Agent Model

Taisei Kaizoji[1,*]

Department of Economics, University of Kiel,
Olshausenstr. 40, 24118 Kiel, Germany
kaizoji@icu.ac.jp
http://www.bwl.uni-kiel.de/vwlinstitute/gwrp/german/team/kaizoji.html

**Abstract.** This paper proposes a model of a stock market which is composed of three different types of traders: fundamentalists, chartists, and noise traders. We investigate the speculative price dynamics though the simulation, and show that a *non-stationary chaos* that is considered as speculative bubble is caused by the heterogeneity of traders' strategies, and furthermore that the distributions of stock returns generated from our heterogeneous agent model have the fat tails which is a stylized fact observed in almost all the financial markets.

## 1 Introduction

A number of studies have found that the empirical distributions of relative price changes (returns) in almost of financial data such as stock prices and exchange rates posse higher peaks around the mean as well as fatter tails than the Normal distribution. This phenomenon is called as *fat tail phenomenon* [1]. Recently several authors have attempted to explain the stylized facts including the fat tails by demonstrating that their interacting heterogeneous agent models can generate the artificial time series with the feature [c.f. Brock and Hommes (1999), Gaunersdorfer and Hommes (2000), Lux (1998), Lux and Marchesi (1999)]. In the models cited above the dynamical systems that describes financial markets have chaotic attractors, and the asset prices fluctuate irregularly around the fundamental values. They reported that the mechanisms by which the chaotic attractors are created, more generally, the bifurcation phenomena the dynamical systems undergo may help account for the stylized facts such as the fat tails and clustering volatility.

In this paper we concerned with speculative bubbles that is an interesting phenomenon that is often observed in the financial markets. The speculative bubbles has often been touched in the rational expectations literature but little attention has been given to this in the recent heterogeneous agent literature. To this end we propose a simple heterogeneous agent model in which speculative

* On research leave from Division of Social Science, International Christian University, Osawa, Mitaka, Tokyo, 181-8585, Japan.

[1] Pagan (1996) provides a excellent survey of the stylized facts on financial markets.

G. Dorffner, H. Bischof, and K. Hornik (Eds.): ICANN 2001, LNCS 2130, pp. 775–781, 2001.

bubbles can be caused by the heterogeneity of traders' trading strategies, and show the distributions of the relative price changes generated from the model have fat tails.

## 2 The Model

We think of a stock market in which a large number of agents trade a stock. The stock market is composed of three groups of traders having the different trading strategies; *fundamentalists*, *chartists* and *noise traders*. Traders can either invest in money or in the stock. We consider the trade in a very short term. Thus, we omit the dividend and the interest rate.

### 2.1 Fundamentalists

We assume that fundamentalist maximizes the utility function, $U(x_t^f, y_t^f) = a(y_t^f + p_{t+1}^f x_t^f) + b(x_t^f - (1 + x_t^f)\log(1 + x_t^f))$, subject to the budget constraint: $y_t^f + p_t x_t^f = 0$, where $x_t^f$ and $y_t^f$ represent the fundamentalist excess demands for the stock, and for money at period $t$. $p_t$ denotes the stock price at period $t$. $p_{t+1}^f$ denotes the expected price by fundamentalists for the next period $t+1$. The fundamentalists are assumed to anticipate the price for the next period $t+1$ using $p_{t+1}^f = p_t + \nu(p^* - p_t)$, that is, the fundamentalists believe that the price will move in the direction of the fundamental price $p^*$, so that they correct their expectation by a factor $\nu$. The problem are solved for the fundamentalist excess demand function for the stock by

$$x_t^f = \exp(\alpha(p_{t+1}^f - p_t)) - 1, \quad \alpha = \frac{a}{b} > 0. \tag{1}$$

This excess demand function means the following: if the stock price $p_t$ is below the expected price $p_{t+1}^f$, then fundamentalists try to buy the stock until the stock price $p_t$ is equal to the expected price, because he thinks that the stock is undervalued, and vice versa.

### 2.2 Chartists

The behavior of a chartist is formalized as maximizing the utility function, $V(x_t^c, y_t^c) = a(y_t + p_{t+1}^c x_t^c) + b(x_t^c - (1 + x_t^c)\log(1 + x_t^c))$ subject to the budget constraint $y_t^c + p_t x_t^c = 0$ where $x_t^c$ and $y_t^c$ represent the chartist excess demand for the stock and for money at period $t$, and $p_{t+1}^c$ denotes the stock price for the next period expected by chartist. The chartist utility function is identical to the fundamentalist one utility function except for the term on the expected price. The chartist excess demand function for the stock is

$$x_t^c = \exp(\alpha(p_{t+1}^c - p_t)) - 1, \quad \alpha = \frac{a}{b} > 0. \tag{2}$$

It follows from this excess demand function that chartists try to buy the stock when they anticipate that the stock price will rise over the next period, and to sell the stocks when they expect the stock price will fall over the next period. Chartists forecast the price for the next period $p^c_{t+1}$ according to the so-called *adaptive expectation* given by $p^c_{t+1} = p^c_t + \mu(p_t - p^c_t)$ where the parameter $\mu$ is called the error correction coefficient.

### 2.3 Noise Traders

The noise traders decide their behavior on base of noisy information $\epsilon_t$. The behavior of noise trader is formalized as maximizing the following utility function given by $W(x^n_t, y^n_t) = g(y^n_t + (p_t + \epsilon_t)x^n_t) + kx^{n2}_t$ subject to the budget constraint $y^n_t + p_t x^n_t = 0$, where $x^n_t$ and $y^n_t$ represent the noise trader desired excess demand for the stock and for money at period $t$. The noisy information at period $t$ $\epsilon_t$ follows IID. The noise trader excess demand function for the stock is

$$x^n_t = \gamma\epsilon_t, \quad \gamma = \frac{g}{k} > 0 \tag{3}$$

### 2.4 The Adjustment Process of the Stock Price

Let us now consider the adjustment process of the stock price in the stock market. We assume that the existence of a market-maker who mediates the trading. If the excess demand in period $t$ is positive (negative), the market maker raises (reduces) the price for the next period $t + 1$. The process of adjustment of the stock price can be rewritten as

$$p_{t+1} - p_t = \theta n[(1 - \kappa - \xi)x^f_t + \kappa x^c_t + \xi x^n_t], \tag{4}$$

where $\theta$ denotes the speed of the adjustment of the price, and $n$, the total number of traders, and $\kappa$ and $\xi$, the proportions of chartists and the noise traders in the total number of traders $n$.

### 2.5 Strategy Switching

The question which we must consider next is how traders choose between the fundamentalist and the chartist strategy. To put it in the model we assume that the proportion of chartist $\kappa$ depends on difference between the squared prediction errors each strategy made. Formally we write the dynamics of the proportion of chartists as follows:

$$\kappa_t = \frac{(1-\xi)}{1 + \exp(\psi(R^c - R^f))} \quad R^c = (p_t - p^c_t)^2, \quad R^f = (p^f_t - p_t) \tag{5}$$

The equation expresses that if the squared prediction error by chartist $R^c$ is greater than the squared prediction error by fundamentalist $R^f$, then some of chartists become fundamentalists, and visa versa.

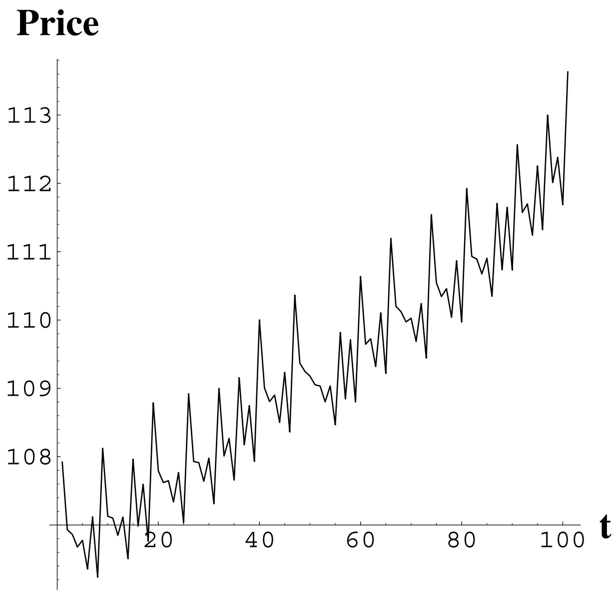


**Fig. 1.** Non-stationary chaos of the stock price

## 3 Dynamics of Speculative Price

It follows that the dynamical system we have seen has the unique equilibrium price which is equal to the fundamental price $p^*$. The global dynamics of the model are quite complex according to the results of the computer simulation to the various values of the parameters. In order to illustrate a typical result from the model put the parameters below as follows: $\alpha = 6$, $\theta = 0.001$, $n = 1000$, $\mu = 0.5$, $\nu = 0.5$, $\gamma = 1$, and $\psi = 0.01$. For a while we also assume that there is no noise trader, that is, $\xi = 0$, and the fundamental price is constant, i.e. $p^* = 10$, so that the price dynamics become deterministic. Figure 1 and Figure 2 indicate typical time series of the price and of the price change obtained from the model with the above set of the parameters. As we see in Figure 1, the dynamic behavior of the stock price is *non-stationary chaos*. Namely, the time series have a upward trend, and fluctuates irregularly around the trend. We may consider the non-stationary chaos as *speculative bubble* that have often been observed in the actual financial markets. On the other hand the price change fluctuates irregularly within a certain range. Namely, the movement of the price change is stationary. Results of the simulation experiments show that continuous increases of $\alpha$ lead the dynamics of the price changes to chaos through the well-known period-doubling bifurcation[2].

[2] For a detailed analysis of the non-stationary chaos see Kaizoji (2001).

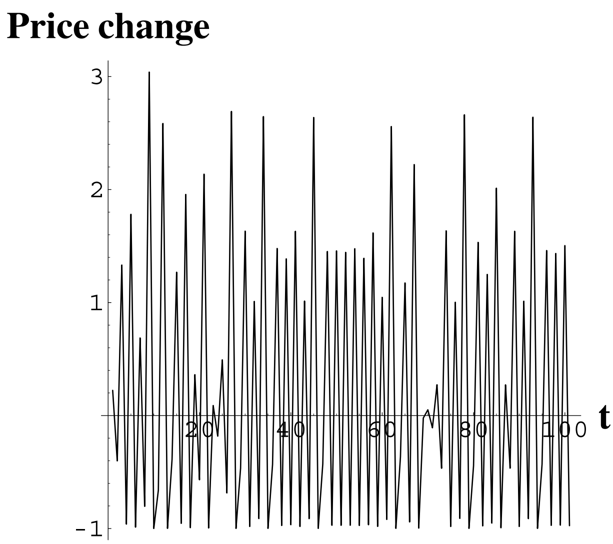


**Fig. 2.** Stationary chaos of the stock price change

**Table 1.** Statistics of return from artificial data

| $\alpha$ | *Mean* | *S.D.* | *Skewness* | *Kurtosis* | *Price* | *Return* | $\rho$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0.014 | 0.023 | 0.061 | stable | stable | 4.83 |
| 2 | 0 | 0.016 | 0.210 | 0.323 | stable | stable | 4.14 |
| 3 | 0 | 0.02 | 0.703 | 1.552 | periodic | periodic | 3.2 |
| 4 | 0 | 0.026 | 1.411 | 4.277 | periodic | periodic | 2.32 |
| 5 | 0 | 0.004 | 2.963 | 17.605 | bubble | chaotic | 4.58 |

### 3.1 Fat Tail Phenomenon

To bring our model close to the actual stock markets let us here assume the existence of noise traders. We put $\xi = 0.3$ below. A number of empirical studies on financial markets have found that the empirical distributions of relative price changes (the so called returns) in almost all financial data show uni-modal bell shapes, and posse excessive fourth moments, so called *leptokurtosis*, (cf. Pagan (1996)). Figure 3 indicates the distribution of the stock returns[3] generated from the model with the above setting of the parameters. The distribution has an uni-modal bell-shape. Table 1 presents sample statistics of stock return. The mean value is very close to zero. The standard deviation is small. Skewness is positive, and kurtosis is significantly positive. The important point to note is that the return distribution exhibits leptokurtosis apparently.

The recent literature finds that the distribution of returns is not only leptokurtotic, but also belongs to the class of *fat tailed* distributions. This finding is quite universally accepted as an universal characteristic of practically all fi-

[3] The stock return is defined as $r_t = \log p_t - \log p_{t-1}$.

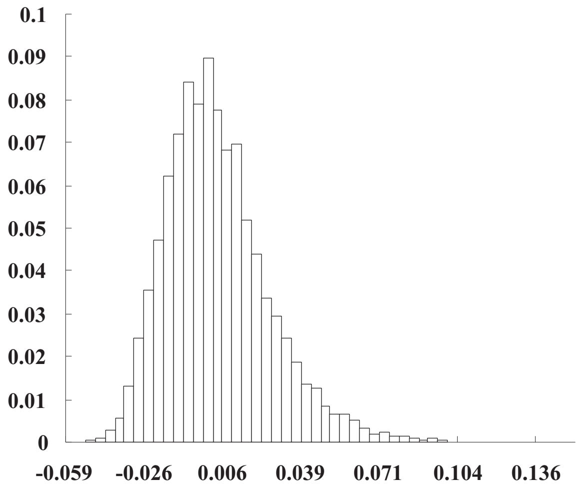


**Fig. 3.** The distribution of the stock returns

nancial returns. More formally it has been shown that the tails of the return distribution follow approximately a power law: $F(|z_t| > x) \sim cx^{-\rho}$, where $F(\dot{})$ denotes the cumulative probability distribution of the normalized stock returns. Most of empirical studies show estimates of the *tail index* $\rho$ falling in the range 2 to 4 (Pagan (1996)). The last column of the table 1 shows the values of $\rho$ of the return distributions generated by our model under the various values of $\alpha$[4]. The values of $\rho$ for tail sizes up to 5 per cent of the size of the data fall in the range 2 to 4 in the cases that periodic solutions are observed in the artificial stock market.

## 4 Concluding Remarks

In this paper we propose a heterogeneous agent model of a stock market. We showed that the non-linearity of the excess demand functions derived from the traders' optimization behavior might generate non-stationary chaos that is regarded as speculative bubbles. Furthermore we showed that the return distributions derived from the model share important characteristics of the empirical financial data: they exhibit leptokurtosis as well as fat tails.

## Acknowledgements

I would like to thank T. Aoki, S-H. Chen, J. Holyst, A. Matsumoto, T. Misawa, T. Onozaki, P. Flaschel R. Frank, W. Zemmler, and T. Asada for helpful sug-

[4] The values of $\rho$ are computed by the Hill tail index estimate that is standard work tool for estimation of the Pareto exponent of the tails.

gestions and comments, and the Alexander Humboldt Foundation for financila support.

## References

1. Brock, W. A., and Hommes, C. H.: Heterogeneous beliefs and routes to chaos in a simple asset pricing model, Journal of Economic Dynamics and Control, 22 (1998) pp.1235-1274.
2. Gaunersdorfer, A., and Hommes, C. H.: A Nonlinear Structural Model fro Volatility Clustering, Working Paper No. 63, Adaptive Information System and Modelling in Economics and Manegement Science (2000).
3. Kaizoji, T.: Speculative Bubbles and Fat Tail Phenomena in A Heterogeneous Model, forthcoming into International Symposia in Economic Theory and Econometrics, Cambridge University Press (2001).
4. Lux, T.: The socio-economic dynamics of speculative markets: interacting agents, chaos and the fat tails of return distributions, Journal of Economic Behaviour and Organization, 33 (1998) pp. 143-165.
5. Lux, T., and M. Marchesi, M.: Volatility Clustering in Financial Markes: A Micro-Simulation of Interacting Agents, International Journal of Theoretical and Applied Finance 3 (2000) pp. 675 - 702.
6. Pagan, A.: The Econometrics of Financial Markets, Journal of Empirical Finance, 3 (1996) 15-102.