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# A model of international financial crises

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**Abstract**

This paper proposes a model of international financial crises that is based on the statistical mechanics. In our model the international stock market is composed of two groups of traders mutually influencing each other with respect to their decision behavior, and *financial contagion* between markets occurs as a result of attempts by traders in the domestic market to imitate the behavior of traders who participate into exchange in a foreign market. This provides a channel through which a crisis in one market such as contemporaneous stock market crashes can be transmitted to other markets. We show that the model can explain the stylized facts characterizing periods of recent international financial crises. © 2001 Published by Elsevier Science B.V.

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## 1. Introduction

In the last two decades the international financial markets has been distinguished by a series of severe financial and currency crises: the U.S. stock market crash of 1987; the Exchange Rate Mechanism (ERM) attacks of 1992; the Mexican peso collapse of 1994; The East Asian crisis of 1997; the Russian collapse of 1998; and the Brazilian Devaluation of 1999. The IT revolution, the worldwide deregulation on the capital transaction and the globalization of economy result in strengthening the instability of international financial markets. These increasingly frequent crises have attracted the attention of academics and policy makers. Of particular interest is why the stock markets around the world fell simultaneously and with surprising uniformity despite widely differing economic circumstances. The nature of the international transmission of the volatility of stock market returns in the periods of recent financial crises has been

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a focus of extensive studies: [1,2] for the 1987 U.S. stock market crash; [2] [5] for 1997 East Asian Crisis; [6] for the 1994 Mexican peso crisis; [7] for the 1992 ERM crisis and the 1998 Russian/Brazilian crisis. These empirical studies report several stylized facts characterizing in periods of international financial crises:

(1) Simultaneous and sharp falls in stock prices tend to concentrate in period of international financial crises.

(2) Volatility of stock market returns increases sharply during crisis period.

(3) Covariance between stock market returns increases sharply during crisis periods.

The first stylized fact says that financial crises originating in one country have often spread internationally, and many stock markets fell together despite widely differing economic circumstances. The second stylized fact is well known as the clustering volatility of stock market returns. A market tends to be volatile violently during crisis period, and then relatively calm for the following several weeks (cf. [8,9]). The third stylized fact states that cross market covariance of stock market returns would be an increasing function of volatility of stock market returns. An implication of these empirical findings is that an increase in stock return volatility in one market could be self-reinforcing and be expected to lead to increase in volatility in the other markets.

The fact that stock markets in different countries are correlated is, of course, not surprising in itself. Those economies are related through trade and investment, so that any news about economic fundamentals in one country most likely has implications for the other country. Any standard asset pricing model such as the international asset pricing models (cf. [10–12]) stresses the role of common fundamental factors to interpret the data solely within a traditional equilibrium framework with fully informed agents. However, these models seem to be unable to account for important characteristics observed in the periods of the financial crises such as the sharp fall in stock prices and excess co-movements of stock returns. Why does stock prices in one country affect the changes in the other countries beyond what is conceivable by connections through economic fundamentals? A possible and convincing explanation for the international correlation of stock returns is *market contagion*. There is an interesting study that presents some empirical evidence about financial contagion during the 1987 U.S. stock market crash [13,14]. In questionnaire surveys Shiller asked U.S. and Japanese institutional investors to recall what they thought and did during the worldwide stock market crash in 1987, and found a remarkable similarity between Japanese and U.S. institutional investors in a number of attitudinal and behavioral dimensions. In particular they showed that both Japanese and U.S. investors had interpretations of the crash that were broadly similar in the importance given to investor psychology rather than fundamentals, and furthermore that many traders in the Tokyo market recall that the day after Black Monday in the New York market they sold stocks on information about the market crash in New York. Under this market-contagion scenario, price movements driven by a traders' herding behavior may be transmittable across borders.

[2] For discussions about cause for large stock market crashes in different angles, see Refs. [3,4].

In this paper we construct a model of international financial markets that is composed of two groups of traders mutually influencing each other with respect to their decision behavior. In our model the market contagion occurs as a result of attempts by traders in the domestic market to imitate the behavior of traders who participate into exchange in a foreign market. This provides a channel through which a crisis in one market such as a large market crash can be transmitted to other markets. We show that the interacting agent model can explain the stylized facts regarding the transmission of shocks across stock markets observed in the periods of the recent international financial crises. To this aim we will reformulate and extend an Ising-like stock market model [3] proposed by Chowhury and Stauffer [17] and Kaizoji [18] into the case of two countries, both with their own stock market. [4] The paper is organized as follows. Section 2 set out the theoretical framework. Section 3 presents a theoretical explanation of the stylized facts about stock market returns across stock markets. In Section 4 the numerical experiments are performed to substantiate the theoretical results. Section 5 give concluding remarks.

## 2. The model

Consider two large stock markets such as the Tokyo and the New York markets. The two stock markets are indexed by $A$ and $B$. In each market a stock market index is traded such as Dow Jones Industrial Average or Nikkei Average. The index of stock market $A$ (stock market $B$) is denoted by $p_A$ ($p_B$). Large numbers of traders participate into trading of each market. Stock market $A$ (stock market $B$) consists of $N_A$ traders ($N_B$ traders). The numbers $N_A$ and $N_B$ are assumed to be constant. Traders in stock market $A$ (traders in stock market $B$) are indexed by $i = 1, 2, \dots, N_A$ ($j = 1, 2, \dots, N_B$). Each of them can share one of two investment attitudes, buy or sell, and buys or sells the fixed amount of the indices $q$ in a period of trading. The investment attitude of trader $i$ (trader $j$) which is represented by $x_i$ ($y_j$) is defined as follows: if trader $i$ (trader $j$) is the buyer of the index at a period, then $x_i = +1$ ($y_j = +1$). If trader $i$ (trader $j$), in contrast, is the seller of the index at a period, then $x_i = -1$ ($y_j = -1$). Thus the analogy between the Ising spin model and the model under study consists in identifying each trader with the spin variables $x_i$ and $y_j$.

### 2.1. Decision-making by traders

The question that we must consider next is how a trader decides his attitude. First, shifts in investment attitude of the other traders can play a major role in a trader's decision. As we have shown in our previous study [18], a trader, who expects a certain exchange profit through trading in a short period, tries to know the total number of

[3] There is a growing literature on modeling of financial markets based upon statistical physics (cf. [15,16]).
[4] The statistical model we here consider is originally proposed by Weidlich and Haag [19] in order to study the structure of social groups of individuals mutually influencing each other with respect to their decision behavior. For an application of the theory of Weidlich and Haag to a financial market see Ref. [20].

the buyers and of the sellers in the market the trader participates into trading in order to infer the local market mood. This factor is often called *bandwagon effect*. Secondly the average opinion of traders in the foreign market might play a role in a trader's decision behavior in the domestic market. This is regarded as a variable to represent an effect of international market psychology on the local market psychology. We call this *market contagion effect*. Thirdly the decision-making of traders will be also influenced by economic fundamentals because it is thought that the stock price reflects fundamentals. Fundamentals are of two types, global and local. A shock to global fundamentals influences traders' investment attitude in both the domestic and foreign markets. On the other hand a shock to local fundamentals affects only the traders' investment attitude in the domestic market. Let us consider that the dollar depreciates against the yen and European currencies. This depreciation most likely affects stock returns in New York, since exporters will benefit from either price competitiveness or a higher profit margin. The news of dollar depreciation (yen appreciation) will be taken into account in Japanese traders' investment attitude, since it will adversely affect Japanese exporters but will favorably affect Japanese importers. This is an example of a shock to global fundamentals. As examples of the other examples of shocks to global fundamentals, a rise in international interest rate, a contraction in the international supply of capital, or a decline in international demand for commodities will be considered because these shocks could slow growth in a number of countries at the same time. Suppose, in contrast, a financial scandal in a country. This shock may lower stock returns in the country. This is an example of a shock to local fundamentals. For simplicity of analysis we assume that the local fundamentals are constant and zeros over time. [5] Finally the traders' decision making is influenced by noises brought on by uncertainty that exists in the markets. The important question is how to represent the trader's decision behavior influenced by noises in a mathematically tractable manner. The traditional method for doing so in the statistical physics literature is to introduce a probabilistic mechanism in the trader's decision-making.

We first introduce the following functions on the traders who exchange in stock market $A$ and who exchange in stock market $B$, $u(x,y)=(\alpha_{11}x+\alpha_{12}y+\beta_1 f)$, and $v(x,y)=(\alpha_{21}x+\alpha_{22}y+\beta_2 f)$ where $x$ and $y$ denote the mean values of the investment attitude $x_i$ and $y_j$, that is, $x=\sum_{i=1}^{N_A} x_i/N_A$ and $y=\sum_{j=1}^{N_B} y_j/N_B$, $f$, the global fundamentals, $\alpha_{11}$ and $\alpha_{22}$, a measure of the strength of the mutual influence between traders in the domestic market. In other words these coefficients represent the strength of the so called bandwagon effect in the domestic market. The coefficients, $\alpha_{12}$ and $\alpha_{21}$ denote a measure of the strength of mutual influence between the traders in the domestic market and the traders in the foreign market. These coefficients represent the strength of the contagion effect of a foreign market on a domestic market. The coefficients, $\beta_1$ and $\beta_2$, denote a measure of the strength of the cross-market linkages. In the language of the spin models, the first term denotes the internal field, and the second and third terms, the external field. Hereafter we suppose that the

[5] The previous work [18] studied on the effect of shocks to local fundamentals on the domestic stock market.

bandwagon and contagion effects, and the cross-market linkages are positive. Namely we assume that $\alpha_{ij}, \beta_i > 0$ $(i, j = 1, 2)$. Using the functions, $u(x, y)$ and $v(x, y)$ we define the transition probabilities for a trader. The transition probabilities for a trader, who trades in stock market $A$, of changing from the seller $-1$ to the buyer $+1$, and vice versa, is defined by $V_{+-}(x, y) = \exp(u(x, y))/\exp(u(x, y)) + \exp(-u(x, y))$, and $V_{-+}(x, y) = \exp(-u(x, y))/\exp(u(x, y)) + \exp(-u(x, y))$. Similarly using $v(x, y)$ the transition probabilities for a trader who trades in stock market $B$ of changing from the seller $-1$ to the buyer $+1$, and vice versa, is written as $W_{+-}(x, y) = \exp(v(x, y))/(\exp(v(x, y)) + \exp(-v(x, y))$ and $W_{-+}(x, y) = \exp(-v(x, y))/\exp(v(x, y)) + \exp(-v(x, y))$. These transition probabilities assume that if the majority opinion of traders in both the market is 'buy', or if global fundamentals turns for the better, then the transition probability from the seller to buyer increase, and in contrast, the transition probability from the buyer to the seller decreases.

The Fokker-Planck equation [6] for the probability distribution $f(x, y; t)$ over two variables $x$ and $y$, using the above transition probabilities, is obtained as

$$\frac{\partial f(x, y)}{\partial t} = -\frac{\partial}{\partial x}[K_x(x, y) f(x, y; t)] - \frac{\partial}{\partial y}[K_y(x, y) f(x, y; t)] + \frac{1}{N_A}\frac{\partial^2}{\partial x^2}[Q_x(x, y) f(x, y; t)] + \frac{1}{N_B}\frac{\partial^2}{\partial x^2}[Q_y(x, y) f(x, y; t)] \quad (1)$$

which is normalized by means of $\int_{-1}^{+1}\int_{-1}^{+1} \mathrm{d}x\,\mathrm{d}y f(x, y; t) = 1$, and where

$$K_x(x, y) = \tanh(u(x, y)) - x, \quad K_y(x, y) = \tanh(v(x, y)) - y\,,$$

$$Q_x(x, y) = 1 - x\tanh(u(x, y)), \quad Q_y(x, y) = 1 - y\tanh(v(x, y))\,.$$

Assuming a distribution $f(x, y; t)$ with only one sharp peak around the mean value $\langle x\rangle$ of $x$ and $\langle y\rangle$ of $y$, the approximate equation for the mean values are obtained as [7]

$$\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} \approx K_x(\langle x\rangle, \langle y\rangle), \quad \frac{\mathrm{d}y}{\mathrm{d}t} \approx K_y(\langle x\rangle, \langle y\rangle)\,. \quad (2)$$

Next by expanding $K_x(x, y)$, $K_y(x, y)$, $Q_x(x, y)$ and $Q_y(x, y)$ in Taylor series around $(\langle x\rangle, \langle y\rangle)$ and considering only the lowest terms of the expansions, the approximate equations for the variances, and covariance are obtained as

$$\frac{\mathrm{d}\sigma_{xx}}{\mathrm{d}t} = \frac{1}{N_A}Q_x(\langle x\rangle, \langle y\rangle) + 2\sigma_{xx}\frac{\partial}{\partial\langle x\rangle}K_x(\langle x\rangle, \langle y\rangle) + 2\sigma_{xy}\frac{\partial}{\partial\langle y\rangle}K_x(\langle x\rangle, \langle y\rangle)\,, \quad (3)$$

$$\frac{\mathrm{d}\sigma_y y}{\mathrm{d}t} = \frac{1}{N_B}Q_y(\langle x\rangle, \langle y\rangle) + 2\sigma_{yy}\frac{\partial}{\partial\langle y\rangle}K_y(\langle x\rangle, \langle y\rangle) + 2\sigma_{xy}\frac{\partial}{\partial\langle x\rangle}K_y(\langle x\rangle, \langle y\rangle)\,, \quad (4)$$

[6] The Fokker-Plank equation is derived from the master equation for the probability distribution $f(x, y; t)$ by using the standard procedure.

[7] For a further details of the procedures leading from the Fokker-Plank equation (or the master equation) to the mean value equations of $x$ and of $y$, see Ref. [19].

$$\frac{\mathrm{d}\sigma_{xy}}{\mathrm{d}t} = \sigma_{xx}\frac{\partial}{\partial\langle x\rangle}K_y(\langle x\rangle,\langle y\rangle) + \sigma_{xy}\left[\frac{\partial}{\partial\langle x\rangle}K_x(\langle x\rangle,\langle y\rangle) + \frac{\partial}{\partial\langle y\rangle}K_y(\langle x\rangle,\langle y\rangle)\right] + \sigma_{yy}\frac{\partial}{\partial\langle y\rangle}K_x(\langle x\rangle,\langle y\rangle)\,, \tag{5}$$

and the variances and covariance are defined by $\sigma_{xx} = \langle(x-\langle x\rangle)^2\rangle$, $\sigma_{yy} = \langle(y-\langle y\rangle)^2\rangle$, and $\sigma_{xy} = \langle(x-\langle x\rangle)(y-\langle y\rangle)\rangle$.

### 2.2. The price adjustment processes

Since traders are supposed to either buy or sell a fixed amount of the stock market index ($q$) in a time, the excess demands for the stock market indices are given by $qxN_A$ and $qyN_B$. We, here, assume the existence of a market-maker whose function is to adjust the stock price in each market. If the excess demands are positive (negative) at a certain time, the market maker raises (reduces) the price for the next time. More precisely, we assume that the rates of the price changes, that is, the stock market returns are calculated as a linear functions of the excess demands:

$$\frac{\mathrm{d}p_A}{\mathrm{d}t} = \lambda qxN_A, \quad \frac{\mathrm{d}p_B}{\mathrm{d}t} = \lambda qyN_B\,, \tag{6}$$

where $\lambda$ represents the speed of adjustment of the stock market price. $p_A$ and $p_B$ denote the logarithm of the stock market price, so that the price adjustment equations (6) represent the stock market returns at a certain time. What has to be noticed is that the mean value equations of $x$ and $y$ (2) represent the time development of the mean values of the stock market returns normalized as $(\mathrm{d}p_A/\mathrm{d}t)/\lambda qN_A$ and $(\mathrm{d}p_B/\mathrm{d}t)/\lambda qN_B$. We hereafter think of $\langle x\rangle$ and $\langle y\rangle$ as proxies for the mean values of the stock market returns in the domestic and foreign market, respectively.

## 3. Structural analysis of the international stock market

The aims of this section is (i) to analyze the qualitative properties of the model of international stock markets proposed in the preceding section, and (ii) to understand theoretically the mechanism of international transmission of financial shocks.

### 3.1. The mean values

We will begin by considering the existence of and the stability conditions for the equilibrium points of the mean value equations (2). An equilibrium state is given by a pair $(\langle x\rangle,\langle y\rangle)$ of solution of the simultaneous equations

$$\phi_A(\langle x\rangle,\langle y\rangle) = \tan h(u(\langle x\rangle,\langle y\rangle)) - \langle x\rangle = 0\,,$$

$$\phi_B(\langle x\rangle,\langle y\rangle) = \tan h(v(\langle x\rangle,\langle y\rangle)) - \langle y\rangle = 0\,.$$

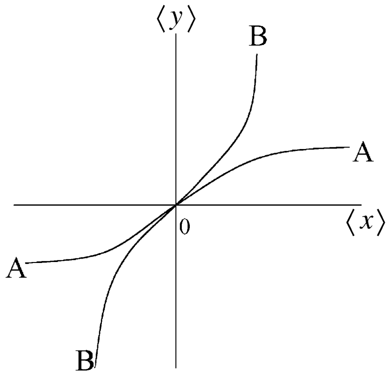


Fig. 1. Stock markets of the type I.

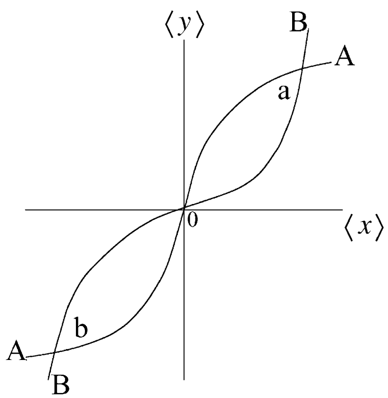


Fig. 2. Stock markets of the type II.

For convenience of analysis we assume that global fundamentals $f$ are constant and zero for a short while. When the points satisfying $\phi_A(\langle x\rangle,\langle y\rangle)=0$ are plotted on the $\langle x\rangle-\langle y\rangle$ plane, where $\langle x\rangle$ is the abscissa and $\langle y\rangle$ is the ordinate, a curve, called curve A, is obtained. Similarly, the points satisfying $\phi_B(\langle x\rangle,\langle y\rangle)=0$ constitute a curve, called curve B. Intersections of these two curves will be called the equilibrium curves, and their intersections denote the equilibrium states.

The features of the equilibrium curves are summarized as follows. When $\alpha_{11}<1$ curve A is continuous and monotonically increasing, symmetric with respect to the origin, and is defined within an interval of $\langle x\rangle$, $\langle x\rangle\in[-1,+1]$, diverging to $\infty$ and $-\infty$ when $\langle x\rangle$ tends to the lower and upper bounds, respectively. (See the curve AA in Figs. 1 and 2.) When $\alpha_{22}<1$, curve B is continuous, monotonically increasing, symmetric with respect to the origin, and bounded within an interval of $\langle y\rangle$, $\langle y\rangle\in[-1,+1]$. See the curve BB in Figs. 1 and 2. In contrast when $\alpha_{11}>1$ ($\alpha_{22}>1$), curve A (curve B) consists of three continuous pieces in which the middle piece is monotonically decreasing and the others are monotonically increasing, and intersects at three points with the abscissa (the ordinate). It is shown from the figures of the equilibrium curves that three types of markets, I, II, and III, are obtained. As in Fig. 1 when the curves intersect at the origin only, the market is called type I. The origin is called the *fundamental equilibrium* at which there is equal numbers of traders sharing both investment attitudes, and both the stock markets are cleared on average. On the other hand when the equilibrium curves intersect at three points, as in Fig. 2, the market is called type II. The two new equilibrium points in Fig. 2 are called a *bull market equilibrium* (the point $a$ in Fig. 2) and a *bear market equilibrium* (the point $b$ in Fig. 2). At a bull (bear) market equilibrium more than half number of traders is the buyer (the seller) in each market, so that simultaneous appreciations (simultaneous depreciations) of the stock returns occur in both the market on average. When $\alpha_{11}, \alpha_{22} > 1$, the equilibrium curves can intersect at more than three points. The markets with $\alpha_{11}, \alpha_{22} > 0$ are called type III. [8]

Let $d_{A0}$ and $d_{B0}$ be the tangents of curve A and curve B, respectively, at the origin. They are explicitly given by

$$d_{A0} = -\frac{\phi_{Ax}(0,0)}{\phi_{Ay}(0,0)} = \frac{1-\alpha_{11}}{\alpha_{12}}, \quad d_{B0} = -\frac{\phi_{Bx}(0,0)}{\partial\phi_{By}(0,0)} = \frac{\alpha_{21}}{1-\alpha_{22}}$$

where $d_{Ax}(0,0)$ is the derivative of $\phi_A(0,0)$ with respect to $\langle x \rangle$. A market of type I is distinguished from the others by the relation $d_{A0} < d_{B0}$ which is expressed in terms of the coefficients as $(1-\alpha_{11})(1-\alpha_{22}) > \alpha_{12}\alpha_{21}$. When $d_{A0} > d_{B0}$ the market is either a type II or a type III.

As is well known from the theory of differential equations, the stability of an equilibrium state $(\langle x \rangle, \langle y \rangle)$ can be examined by the matrix

$$J = \begin{bmatrix} \phi_{Ax} & \phi_{Ay} \\ \phi_{Bx} & \phi_{By} \end{bmatrix}$$

evaluated at the point. An equilibrium state is stable when the following conditions hold and unstable when at least one of the conditions does not hold: (I) the determinant of J is positive, and (II) the trace of J is negative. The condition (I) is equivalent to $d_A < d_B$ where $d_A$ and $d_B$ are the tangents of curves A and B, respectively. Let us consider a type I market. When the condition (I) is applied to the fundamental equilibrium, the condition $(1-a_{11})(1-a_{22}) > a_{12}a_{21}$ is obtained. When the condition (II) is applied to the fundamental equilibrium, it reduces to $\alpha_{11} + \alpha_{22} < 2$. Therefore in a type I system the fundamental equilibrium is monostable.

Next consider a type II market. For a type II market the condition (I) is easily solved graphically. It follows from properties of the equilibrium curves that the condition (II) holds for the bear market equilibrium and the bull market equilibrium and in contrast, the condition (II) does not hold for the fundamental equilibrium. Thus, the fundamental equilibrium is unstable. We can demonstrate that the condition (II) holds for both of the bear market equilibrium and the bull market equilibrium when $\alpha_{11}, \alpha_{22} < 1$. [9] As

[8] To avoid the mathematical results are complicate unnecessarily we are not concerned here with a market of the type III.

[9] The proof is trivial and omitted.

a result we can say that both the bear and bull market equilibria are stable and in contrast the fundamental equilibrium is unstable in the type II market.

The above results can be summarized as follows: The stock markets have the unique and stable equilibrium when the bandwagon and contagion effects are positive but weak. At the equilibrium that called the fundamental equilibrium there are which there is equal numbers of traders sharing both investment attitudes and so both the stock markets keep demand and supply in balance on average. However, when the contagion effect in the markets becomes strong, the fundamental equilibrium become unstable and two stable equilibrium points, a bull market equilibrium and a bear market equilibrium appear, so that the market changes from a type I to a type II. At the bull (bear) market equilibrium there are excess demands for the stock market indices in both the markets on average, and so contemporaneous appreciations (contemporaneous depreciations) occur. Note that the contemporaneous appreciation or depreciation is not caused by changes of the global fundamentals for the better, but caused by the international market psychology, that is, the strong positive and strong contagion effect and a positive bandwagon effect.

### *3.2. The variances and the covariance*

Next we investigate the existence of and the stability for the equilibrium point of the variances and covariance. The equilibrium point of variances and covariance are defined as a point such that the right hand sides of all the equations for variances and covariance is zero at the same time. We can demonstrate that the equilibrium curves for the variances and covariance necessarily intersect at a point. The unique equilibrium point is

$$\bar{\sigma}_{xx} = \frac{(|J| + \phi_{By}^2)\bar{Q}_x + \phi_{Ay}^2\bar{Q}_y}{-2(\phi_{Ax} + \phi_{By})|J|}, \quad \bar{\sigma}_{yy} = \frac{(|J| + \alpha_{11}^2)\bar{Q}_y + \alpha_{21}^2\bar{Q}_x}{-2(\alpha_{11} + \alpha_{22})|J|}$$

$$\bar{\sigma}_{xy} = \frac{\phi_{By}\phi_{Bx}\bar{Q}_x + \phi_{Ax}\phi_{Ay}\bar{Q}_y}{2(\phi_{Ax} + \phi_{By})|J|},$$

where $|J|$ denotes the determinant of the Jacobian matrix $J$ in the preceding subsection, $\bar{Q}_x = Q_x(x, y)/N_A$, $\bar{Q}_y = Q_y(x, y)/N_B$. The equilibrium point of the variances and covariance is derived by utilizing the equilibrium curves for the mean values. It follows from the definition of $\phi_A(x, y)$ and $\phi_B(x, y)$ that the value of covariance at the equilibrium is positive if $0 < \alpha_{11}, \alpha_{22} < 1$.

Similarly the stability conditions for the equilibrium point $(\bar{\sigma}_{xx}, \bar{\sigma}_{yy}, \bar{\sigma}_{xy})$ are obtained by using the stability conditions for the equilibrium points of the mean values $\langle x\rangle$ and $\langle y\rangle$.

*Stability conditions*: when the stability conditions for the equilibrium point of the mean values (I) and (II) hold, the equilibrium point of the variances and covariance is also stable.[10]

[10] The conditions are easily derived from the standard theory of stability. Thus, we omit the proof.

Using the above results on the statistic of the stationary distribution of $f(x, y)$, we can say the feature of the stationary distribution as follows. For a market of the type I the stationary distribution of $f(x, y)$ has a unique peak around the fundamental equilibrium. On the other hand the stationary distribution of $f(x, y)$ has two maxima around a bull market equilibrium and a bear market equilibrium for a market of the type II.

### 3.3. The international transmission mechanism of global shocks

In this subsection we inquire into the mechanism of the international transmission programmed into the model. Let us introduce the following derivatives. As a beginning, we will examine how an increase in the variance influences time development of the covariance. We differentiate the covariance equation (5) with respect to the variances.

$$\frac{\partial \dot{\sigma}_{xy}}{\partial \sigma_{xx}} = \frac{\alpha_{21}}{\cosh^2(v(\langle x \rangle, \langle y \rangle))} > 0, \quad \frac{\partial \dot{\sigma}_{xy}}{\partial \sigma_{yy}} = \frac{\alpha_{12}}{\cosh^2(u(\langle x \rangle, \langle y \rangle))} > 0\,,$$

where $\dot{\sigma}_{xy}$ denotes derivative of the covariance with respect to time $\dot{\sigma}_{xy}$. $\dot{\sigma}_{xy}$ increases as the variance $\sigma_{xx}$ or $\sigma_{yy}$ increases. Added to this, the stronger the contagion effect denoted by $\alpha_{12}$ or $\alpha_{21}$ becomes, the larger the change in the covariance is when the variances change.

Next, let us consider the inverse problem. How does changes in the covariance affect the time development of the variances? Then the effect of a change in the covariance on the variances may be verified by Differentiating the derivative of the variances with respect to time, $\dot{\sigma}_{xx}$ or $\dot{\sigma}_{yy}$ with respect to the covariance $\sigma_{xy}$, we obtain the following,

$$\frac{\partial \dot{\sigma}_{xx}}{\partial \sigma_{xy}} = 2\frac{\partial \dot{\sigma}_{xy}}{\partial \sigma_{yy}} > 0, \quad \frac{\partial \dot{\sigma}_{yy}}{\partial \sigma_{xy}} = 2\frac{\partial \dot{\sigma}_{xy}}{\partial \sigma_{xx}} > 0\,. \tag{7}$$

$\dot{\sigma}_{xx}$ and $\dot{\sigma}_{yy}$ are positive and increases as the covariance $\sigma_{xy}$ increases. These tell us more, the effect of a change in the covariance on the time derivative of the variance of $x$ is related with the effect of a change in the variance of $y$ on the time derivative of the covariance with respect to time, and vice versa. Therefore an rise in the variance of $y$ increases the time derivative of the variance through an increase in the covariance. As an example if a shock on global fundamentals influences the variance of the stock market return in the foreign market $\sigma_{yy}$ change, then its change affects the variance of the stock market returns in the domestic market $\sigma_{yy}$ through a change in the covariance $\sigma_{xy}$, and vice versa. This is the mechanism of the international transmission of financial shocks in our model. For this mechanism of international transmission, changes in the volatility of the stock return in one market can lead to changes in volatility in the other market. The most important addition to be made to what we have said about the mechanism of the international transmission is that for a stable system such as a type I system the mechanism of international transmission could not appear conspicuously, but when a stable equilibrium becomes unstable for any reason, this international transmission mechanism can be expected to give cause to the excess volatility and the

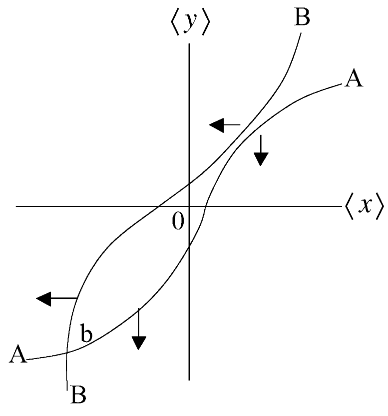


Fig. 3. The market-phase transition.

excess co-movements of the stock market returns in both the markets. We will discuss about this issue in the next subsection.

### 3.4. *Excess volatility, excess co-movement, and contemporaneous crashes*

We consider a market of the type II again. The distribution function $f(x, y)$ has two peaks around a bull market equilibrium and a bear market equilibrium. Suppose that in the initial state both the stock markets are assumed to stay at the stable bull market equilibrium (the point $a$ in Fig. 2). This means that bubbles occur in both the markets at the same time. Then assume that a bad news shock on global fundamentals such as a rise in the international interest rate arrive in all the traders in both the markets. What happen to the markets? To find how bad shocks affect the markets, we take the total differentiation of the equilibrium curve of the mean values $\phi_A(\langle x\rangle, \langle y\rangle) = 0$ and $\phi_B(\langle x\rangle, \langle y\rangle) = 0$. This tells us that the values of the derivatives $\mathrm{d}\langle x\rangle/\mathrm{d}f$ must satisfy

$$\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}f} = \frac{\beta_1/\cosh^2(u(\langle x\rangle, \langle y\rangle))}{1 - \alpha_{11}/\cosh^2(u(\langle x\rangle, \langle y\rangle))} > 0\,, \tag{8}$$

where the value of $\langle y\rangle$ is unchanged. Similarly,

$$\frac{\mathrm{d}\langle y\rangle}{\mathrm{d}f} = \frac{\beta_2/\cosh^2(v(\langle x\rangle, \langle y\rangle))}{1 - \alpha_{22}/\cosh^2(v(\langle x\rangle, \langle y\rangle))} > 0\,, \tag{9}$$

where the value of $\langle x\rangle$ is unchanged. The sign of both the derivatives is positive. This means that bad news on global fundamentals push curve A downward, and curve B leftward. The directions of sifts of curves are illustrated by the arrows in Fig. 3. The bull market equilibrium vanishes when the negative impact of the shock reaches a critical value. In other words one peak of the distribution function $f(x, y; t)$ disappears, and the non-equilibrium distribution becomes uni-modal. A further fall in $f$ give incentive for the traders in both the market to trade and lead to changes of the stock returns. This triggers the phase transition from the bull market equilibrium to the bear market equilibrium. This phase transition is considered as the contemporaneous stock market crashes. (See Fig. 3.)[11] It is not so easy to calculate the critical value of the global fundamentals $f$ so as to give cause to contemporaneous crashes. For simplicity of analysis, consider the case that both the stock markets have the same structure, that is, $\alpha_{11} = \alpha_{22}$, $\alpha_{12} = \alpha_{22}$, and $\beta_1 = \beta_2$. Then the critical value is obtained as $f$ so as to satisfy $\cosh^2[\beta_1 f \pm \sqrt{(\alpha_{11} + \alpha_{12})(\alpha_{11} + \alpha_{12} - 1)}] = (\alpha_{11} + \alpha_{12})$. For the critical value of $f$, the two equilibrium points except the bear market equilibrium coincide at $\langle x \rangle_c = \langle y \rangle_c = \sqrt{(\alpha_{11} + \alpha_{12} - 1)/(\alpha_{11} + \alpha_{12})}$.

Next, let us look closely at the movement of the variances and covariance during the phase transition from a bull market equilibrium to a bear market equilibrium. At the beginning of the crisis, the dynamics of the mean values, the variances, and the covariance become unstable simultaneously buy temporally. The traders become aware of changes of the domestic and foreign market moods, and they begin to revise their investment attitude more drastically. The variances and the covariance are self-reinforced by the mutual influence of the unstable international market psychology. This is the mechanism of the international transmission of shocks that we have seen in the preceding section. As the dynamics of the mean values approaches to the bear market equilibrium, and the non-equilibrium distribution converges to the new stationary distribution, the markets relatively calm.

In this subsection we tried to explain an international financial crisis from a market-phase transition point of view. This seems to gives a good explanation of the stylized facts regarding the transmission of shocks across stock markets, particularly the excess volatility of the variances and excess co-movement of the stock market returns.

## 4. Numerical analysis

In this section numerical experiments are conducted in order to explore furthermore the model's dynamic behavior for a market of the type II. The numerical experiments serve the purpose of numerically substantiating the analytical results presented in Section 3 and of demonstrating phenomenologically that the model can be a useful tool to illustrate the stylized facts characterizing periods of international financial crises.

To examine the effect of shocks on global fundamentals on the stock market returns we assume that change of the global fundamentals is stochastic process that follows the Gaussian random force, that is, $\mathrm{d}f/\mathrm{d}t = \varepsilon$, $\varepsilon \sim N(0, 1)$. We consider a market of the type II with a relatively weak bandwagon effect and a strong contagion effect. We choose the following set of the parameters: $\alpha_{11} = 0.80$, $\alpha_{12} = 1.2$, $\alpha_{21} = 1.5$, $\alpha_{22} = 0.8$, $\beta_1 = 4$, and $\beta_2 = 3$, $\lambda = 0.001$, $q = 1$, and $N_A = N_B = 1000$. Under the set of the parameters the distribution function has the stable two maxima, a bull market equilibrium and a bear market equilibrium, and an unstable bottom. Fig. 4 shows time series of the mean value of the stock market returns, and global fundamentals $f$. Note that $\langle x \rangle$ ($\langle y \rangle$)

[11] It follows that a good shock on the global fundamentals could give cause to the contemporaneous bubbles.

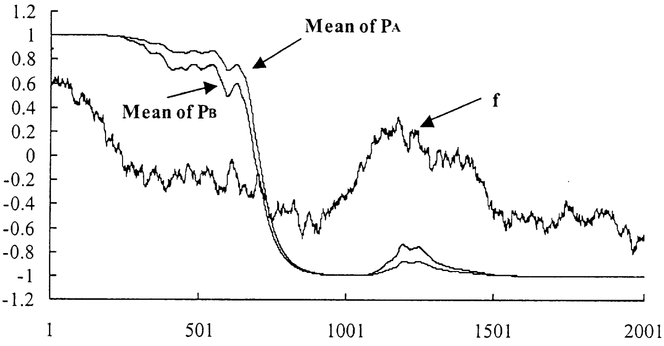


Fig. 4. Time series of the mean value of the stock market returns, and global fundamentals $f$. The values of the parameters are given as $\alpha_{11}=0.80$, $\alpha_{12}=1.2$, $\alpha_{21}=1.5$, $\alpha_{22}=0.8$, $\beta_1=4$, and $\beta_2=3$, $\lambda=0.001$, $q=1$, and $N_A=N_B=1000$.

is in proportion to the mean values of the stock market return, $p_A$ ($p_B$). This figure indicates that when global fundamentals $f$ lower and become negative, $\langle p_A\rangle$ and $\langle p_B\rangle$ start to fluctuate and to lower, and falls sharply and simultaneously. This sharp falls is obviously caused by a phase transition from the bull market phase to bear market phase. Fig. 5a–c show the time developments of the variances and the covariance of the stock market returns, respectively. All the time series have a large and sharp peak in the period of the contemporaneous stock market crash. These figures tell us that when the volatility of the stock market returns is high, the covariance is also high.

## 5. Concluding remarks

The recent international financial crises increased research interest into how financial disturbances transmit from one market to another. We proposes a model based on statistical mechanics to explain the mechanism of international transmission. We may say that our results obtained by the theoretical analyses and the numerical experiment gives a good account of the stylized facts regarding the international transmission of shocks found by the empirical studies on the recent financial crises.

Finally the most important addition to be made to what we have said about our model is that the stronger cross-market linkage measured by $\beta_1$ and $\beta_2$ is, the larger the probability of occurrence of the market-phase transition that we regard as an international financial crisis is. In the last two decades the globalization of economy obviously has reinforced the cross-market linkages. We conjecture that this might be one reason that the international financial markets become vulnerable to shocks.

From these theoretical results we would now like to go on to empirical studies based on our theoretical model on the mechanism of the international transmission of shocks in the periods of the international financial crises.

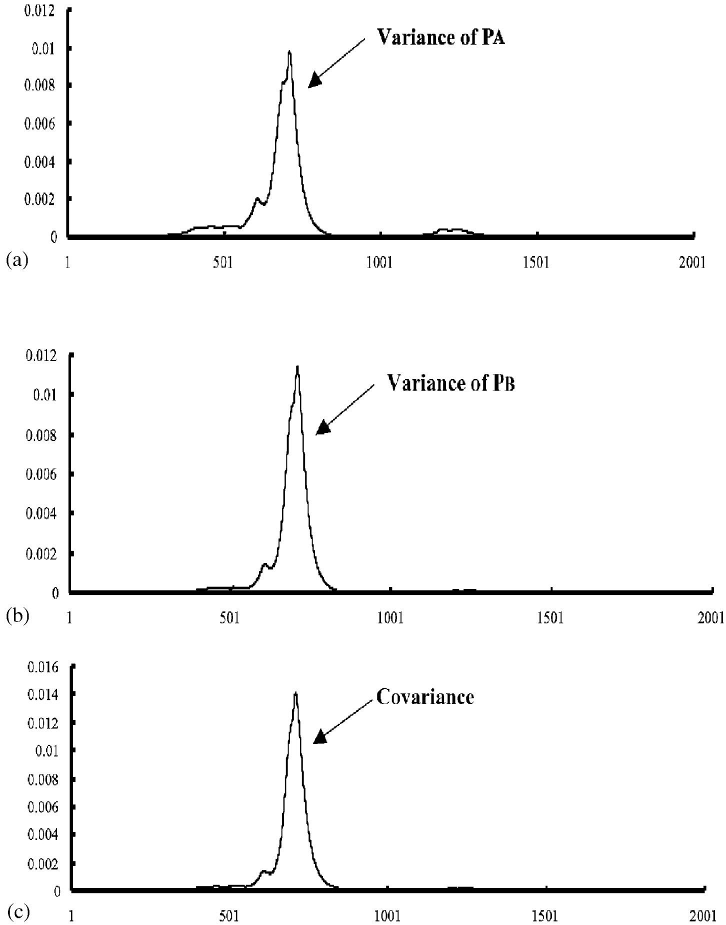


Fig. 5. a–c. The time developments of the variances and the covariance of the stock market returns. The values of the parameters are given as $\alpha_{11} = 0.80$, $\alpha_{12} = 1.2$, $\alpha_{21} = 1.5$, $\alpha_{22} = 0.8$, $\beta_1 = 4$, and $\beta_2 = 3$, $\lambda = 0.001$, $q = 1$, and $N_A = N_B = 1000$.

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