Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 336 (2004) 563 – 570
PHYSICA A
www.elsevier.com/locate/physa

Power law for the calm-time interval of price changes

Taisei Kaizoji[a,b,*], Michiyo Kaizoji[b]

[a] *Division of Social Sciences, International Christian University, Mitaka, Tokyo 181-8585, Japan*
[b] *Econophysics Library, 5-9-7-B Higashi-cho, Koganei-shi, Tokyo 184-0011, Japan*

Received 30 August 2003; received in revised form 22 December 2003

Abstract

In this paper, we describe a newly discovered statistical property of time series data for daily price changes. We investigated quantitatively the *calm-time intervals* of price changes for 800 companies listed in the Tokyo Stock Exchange, and for the Nikkei 225 index in the 27-year period from January 1975 to December 2001. A calm-time interval is defined as the interval between two successive price changes above a fixed threshold. We found that the calm-time interval distribution of price changes obeys a power-law decay. Furthermore, we show that the power-law exponent monotonically decreases with respect to the threshold.

PACS: 89.90.+n; 05.40.Df

Keywords: Econophysics; Stock price changes; Calm-time interval; Power-laws

1. Introduction

Some stylized facts of financial time series are well known to hold true. Examples include (i) fat-tailed distribution [1] of relative price changes, so called log-returns, (ii) for larger values of relative price changes, the power-law distribution of relative price changes [2–9], (iii) volatility clustering [10,11] which is described as on–off intermittency in literature of nonlinear dynamics, (iv) multi-fractality of volatility [12–16], and so on.

* Corresponding author. Division of Social Sciences, International Christian University, Mitaka, Tokyo 181-8585, Japan.
E-mail address: kaizoji@icu.ac.jp (T. Kaizoji).
URL: http://subsite.icu.ac.jp/people/kaizoji/econophysics/

0378-4371/$ - see front matter
doi:10.1016/j.physa.2003.12.054

More recently several authors have investigated statistical properties of waiting times of high-frequency financial data [17–24]. In particular, Enrico Scalas and his co-workers [17–21] have applied the theory of continuous-time random walk (CTRW) to the financial data. They also found that the waiting-time survival probability for high-frequency data of the 30 DJIA stocks is non-exponential [21]. In this study we examined further the statistical properties of the waiting-time of stock price changes. An interesting question occurring in financial markets is: what are the statistical properties of the time intervals between two successive price changes? In particular, how long after one large price fluctuation will the next large price fluctuation occur? To answer these questions, we investigated the distribution of the calm-time interval [21] describing the time interval between two successive price changes above a fixed threshold. We analyzed quantitatively (i) a database covering securities of companies listed into the Tokyo Stock Exchange (TSE), and (ii) the Nikkei 225 index from 4 January 1975 to 28 December 2001. The database covering securities of companies listed into the TSE provides daily data of closing prices and covers the period between 4 January 1975 and 28 December 2001. We selected the 800 companies among all the companies listed that had an unbroken series of daily closing prices for the entire 27-year period. Each time series had approximately 7000 data points, corresponding to the number of trading days in the 27-year period. We report how the calm-time interval distributions of price changes are well approximated by a power-law function. Furthermore, we show that the power-law exponent monotonically decreases with respect to the threshold.

2. The calm-time interval distribution of price changes

We first analyzed the set of data of stock prices for 800 companies covering the 27-year period from 4 January 1975 to 28 December 2001.

We formed 800 time series of $p_j(t)$, which denotes the closing price of company j. We defined price change $S_j(t)$ as increment of the price $S_j(t) = p_j(t) - p_j(t-1)$ where $p(t)$ is the closing price on trading day t. We normalized the price change as follows:

$$s_j(t) = \frac{[S_j(t) - \bar{S}_j]}{V_j}; \quad \bar{S}_j = \frac{1}{T}\sum_{t=1}^{T} S_j(t)\,, \tag{1}$$

where V_j is the standard deviation of company j and $\bar{S}_j(t)$ is a time average. We obtained about 7000 normalized price changes $s_j(t)$ per company in the 27-year period. We first measured the calm-time intervals. To do so, we introduce a threshold θ of the absolute value of normalized price changes, $|s_j(t)|$. A calm-time interval τ_j is accurately defined as the time interval from 1 day that the absolute value of the normalized price change is above the threshold to the next day that it exceeds the threshold. We counted the number of calm-time intervals greater than 1 day, and then made a cumulative distribution of the calm-time intervals of the normalized price changes, $P(|\tau_j| > \tau)$. In this paper we focus attention on statistical properties of the calm-time interval of price change rather than those of the calm-time interval of a return defined as the logarithmic price change. The main reason is that we investigated the statistical properties of the

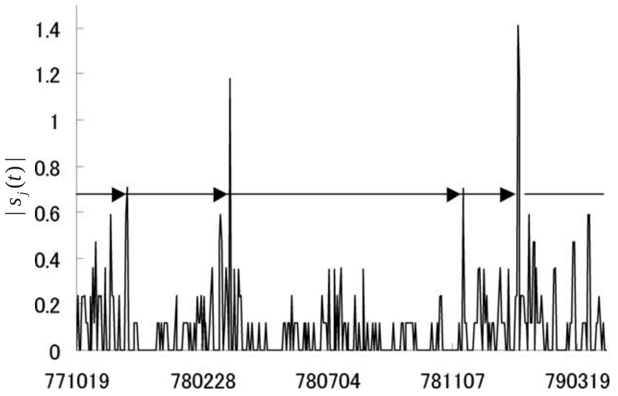

Fig. 1. The time series of the calm-time intervals of price changes. The vertical axis indicates the absolute value of the price increment, $|s_j|$, and the horizontal axis indicates date. The arrows on the figure illustrate the calm-time intervals.

calm-time intervals of the returns but we could not obtain the clear results, comparing to the empirical results of the price changes.

Tomen Corp., a large Japanese business company, serves as a typical example of the 800 large companies we analyzed. Fig. 1 illustrates the calm-time interval of the price changes. The vertical axis indicates the absolute value of the price change, $|s_j|$, and the horizontal axis indicates date. The arrows on the figure illustrate the calm-time intervals. In this case we established the threshold up $\theta = 0.7$. Fig. 2 presents the log–log plots of the cumulative distribution of the calm-time intervals of price changes with the fixed threshold $\theta = 0.7$. The dots represent the observed cumulative distributions, whereas the solid lines represent the power-law distributions, which is expressed by

$$P(x) \equiv P(\tau_{tomen} \geqslant \tau) \sim \frac{1}{\tau^{\alpha}} . \tag{2}$$

Linear regression fit in the region from 1 to 100 standard deviations yields $\alpha = 1.13 \pm 0.02$, $R^2 = 0.99$.

To confirm the robustness of the above analysis, we repeated this analysis for each time series of price changes for each of the 800 companies. For all of the companies, the asymptotic behavior of the functional form of the cumulative distributions was consistent with a power law. Fig. 3, showing the log–log plot of cumulative distributions of the calm-time intervals with the threshold $\theta = 0.7$ for 10 companies selected randomly from the 800 companies, demonstrates that the calm-time intervals of the price changes obey power-law decay. The power-law exponent α is distributed in the range from 1 to 2.[1] Under the threshold $\theta = 0.7$ the estimates of the power-law exponent α were, for all but 4 of the 800 companies, within the stable Lévy domain, $0 < \alpha < 2$.

[1] The estimates of the power-law exponent α were sensitive to the bounds of the regression used for fitting. Thus we used the coefficient of determination R^2 of the linear regression line as a standard to determine the appropriate values of the power-law exponent α. We chose the results of the regression fit where the coefficient of determination was greater than 0.98.

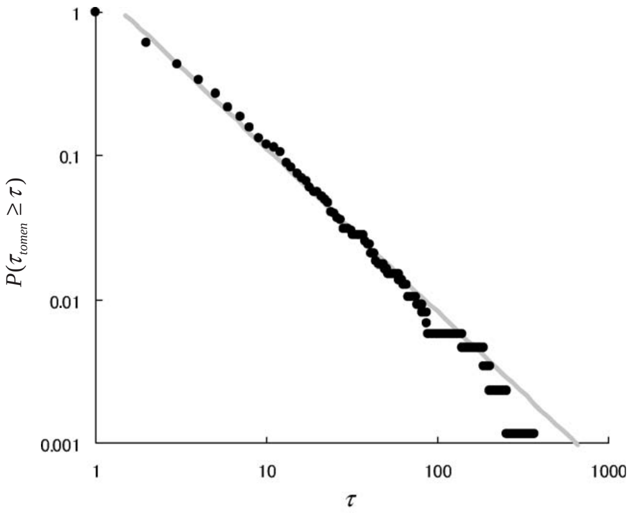

Fig. 2. The log–log plots of the cumulative distribution of the calm-time interval for Tomen Corp. in the 27-year period from 4 January 975 to 28 December 2001 with the threshold $\theta = 0.7$. The dots represent the observed cumulative distributions, whereas the solid lines are of the power-law distributions $P(\tau_{tomen}) \sim \tau^{-\alpha}$ with the exponent $\alpha = 1.13$.

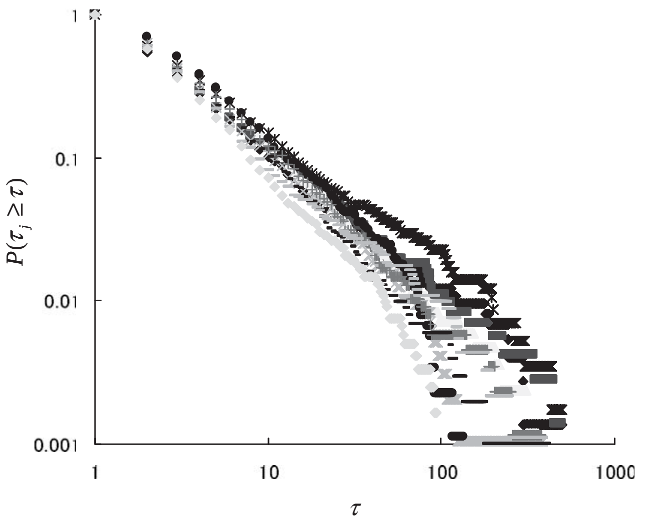

Fig. 3. The log–log plots of the cumulative distribution of the calm-time interval for 10 companies with the threshold $\theta = 0.7$. The 10 companies were selected randomly from among 800 companies listed into the TSE.

Fig. 4 shows the histogram for the power-law exponent α, obtained from regression fits to the individual cumulative distributions of all 800 companies. Fifty-seven percent of all the estimates of α were between 1.3 and 1.5.

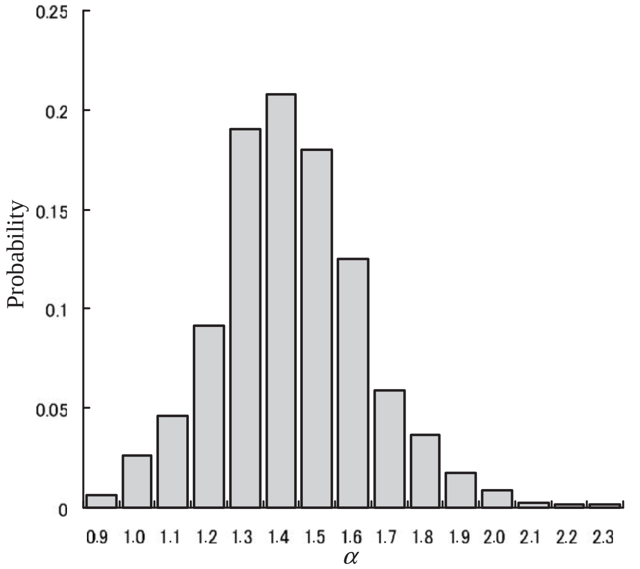

Fig. 4. The histogram of the power-law exponent α estimated from the cumulative distributions of the calm-time intervals for 800 companies listed into the TSE with the threshold $\theta = 0.7$.

We next analyzed the price series data of price series for the Nikkei 225 index in the period from 4 January 1975 to 28 December 2001. The Nikkei 225 index is the Dow–Jones average of 225 industrial stocks listed in the Tokyo Stock Exchange. Fig. 5 presents the log–log plots of the cumulative distributions of the calm-time interval for the Nikkei 225 index for the threshold $\theta = 0.25$. The dots represent the observed cumulative distribution, whereas the solid lines are of the power-law distributions, which are well expressed by $P(\tau_{nikkei} \geqslant \tau) \sim \tau^{-\alpha}$. Regression fits in the region $1 \leqslant \tau \leqslant 90$ yield $\alpha = 1.13 \pm 0.02$, $R^2 = 0.992$.

2.1. Dependency of the power-law exponents on the threshold

Finally, we investigated the dependency of the power-law exponent α on the threshold, θ. Fig. 6 presents the plot of the cumulative distributions of the calm-time intervals for Tomen Corp. under θ ranging from 0.1 to 0.9. Then we investigated the Nikkei 225 index. Fig. 7 shows the plot of the cumulative distributions of calm-time intervals for the Nikkei 225 index under θ ranging from 0.15 to 0.3. Table 1 shows the threshold τ and the corresponding power-law exponent α. One can see that in both the cases the power-law exponent α monotonically decreases with respect to the threshold θ.

Through our extensive examinations using price data for the 800 companies, we have ascertained that this tendency is robust.

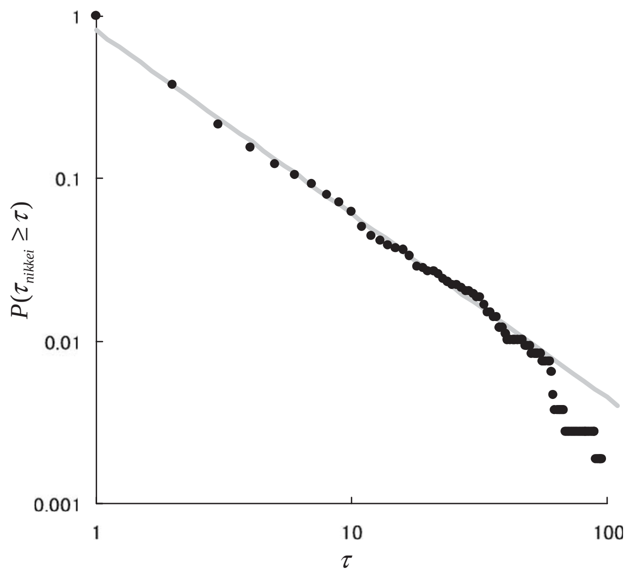

Fig. 5. The log–log plots of the cumulative distribution of the calm-time interval for the Nikkei 225 index in the 27-year period from 4 January 1975 to 28 December 2001 with the threshold at $\theta = 0.25$. The dots represent the observed cumulative distributions, whereas the solid lines represent the power-law distributions $P(\tau_{nikkei}) \sim \tau^{-\alpha}$ with the exponent $\alpha = 1.13$.

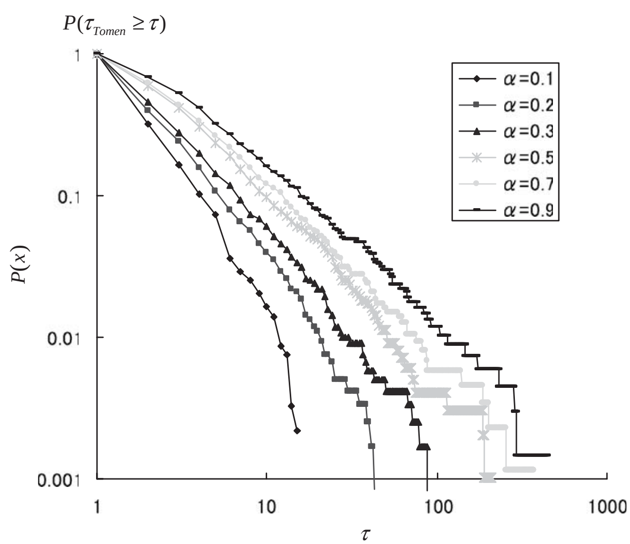

Fig. 6. The plot of the cumulative distributions of the calm-time intervals for Tomen Corp. under thresholds θ ranging from 0.1 to 0.9.

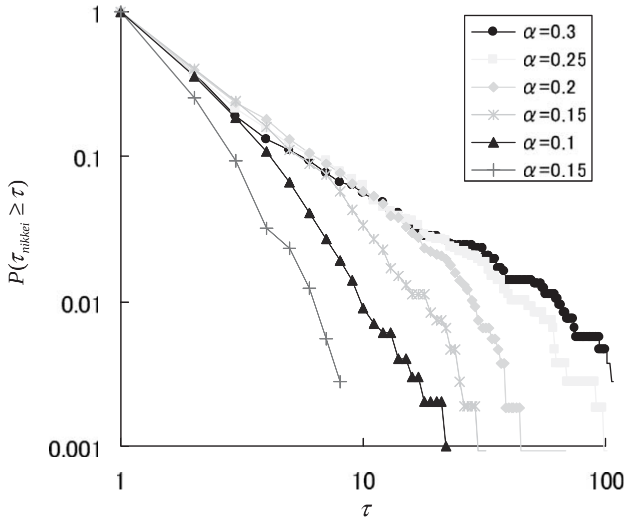

Fig. 7. The plot of the cumulative distributions of the calm-time interval for the Nikkei 225 index under thresholds θ ranging from 0.15 to 0.3.

Table 1
Dependency of the power-law exponent α on the threshold θ

Tomen corp.		The Nikkei 225 index	
Threshold θ	The exponent α	The threshold θ	The exponent α
0.1	1.81	0.05	2.47
0.2	1.43	0.1	2.24
0.3	1.31	0.15	1.66
0.5	1.22	0.2	1.25
0.7	1.13	0.25	1.17
0.9	0.97	0.3	1.16

3. Conclusion

In conclusion, we have discovered a new scale-free trend for calm-time intervals between large fluctuations of daily stock prices. Our results show that the power law of the calm-time interval is very robust. The power-law statistics shown here are not only an interesting theoretical finding but are presumably also a practical tool for measuring the risk of security investments. Our empirical results constitute a condition that any theory of financial markets would have to satisfy. The next step is to model the power laws of calm-time intervals of the lower-frequency financial data. The theoretical study will be left for future work.

Acknowledgements

I would like to thank Enrico Scalas for helpful comments and suggestions. Financial support by the Japan Society for the Promotion of Science under the Grant-in-Aid, No. 06632 is gratefully acknowledged. All remaining errors, of course, are ours.

References

[1] A. Pagan, J. Empirical Finance 3 (1996) 15.
[2] B.B. Mandelbrot, J. Business 36 (1963) 394.
[3] E.F. Fama, J. Business 38 (1965) 34.
[4] K.G. Koedijk, M.M.A. Schafgans, C.G. De Vries, J. Int. Econ. 29 (1990) 93.
[5] T. Lux, Appl. Financial Econ. 6 (1996) 463.
[6] R.N. Mantegna, H.E. Stanley, Nature 376 (1995) 46.
[7] P. Gopikrishnan, M. Meyer, L.A.N. Amaral, H.E. Stanley, Eur. Phys. J. B 3 (1998) 139.
[8] P. Gopikrishnan, V. Plerou, L.A.N. Amaral, M. Meyer, H.E. Stanley, Phys. Rev. E 60 (1999) 5305.
[9] V. Plerou, P. Gopikrishnan, L.A.N. Amaral, M. Meyer, H.E. Stanley, Phys. Rev. E 60 (1999) 6519.
[10] R.F. Engle, Econometrica 50 (1982) 987.
[11] J.Y. Campbell, A.W. Lo, A.C. MacKinlay, The Econometrics of Financial Markets, Princeton University Press, Princeton, NJ, 1997.
[12] T. Lux, Appl. Econ. Lett. 3 (1996) 701.
[13] S. Ghashghaie, et al., Nature 381 (1996) 767.
[14] F. Schmitt, D. Schertzer, S. Lovejoy, Appl. Stochastic Models Data Anal. 15 (1999) 29.
[15] A. Arneodo, J.-F. Muzy, D. Sornette, Eur. Phys. J. B 2 (1998) 277.
[16] J.-P. Bouchaud, M. Potters, M. Meyer, Eur. Phys. J. B 13 (2000) 595.
[17] E. Scalas, R. Gorenflo, F. Mainardi, Physica A 284 (2000) 376.
[18] F. Mainardi, M. Raberto, R. Gorenflo, E. Scalas, Physica A 287 (2000) 468.
[19] M. Raberto, E. Scalas, R. Goren, F. Mainardi, The waiting-time distribution of Liffe bond futures, 2000. arXiv: cond-mat/0012497.
[20] M. Raberto, E. Scalas, F. Mainardi, Physica A 314 (2002) 749–755.
[21] E. Scalas, R. Gorenflo, F. Mainardi, M. Mantelli, M. Raberto, Anomalous waiting times in high-frequency financial data, arXiv: cond-mat/0310305, 2003.
[22] L. Sabatelli, S. Keating, J. Dudley, P. Richmond, Waiting time distribution in financial markets, Eur. Phys. J. B 27 (2002) 273–275.
[23] K. Kim, S.-M. Yoon, Dynamical behavior of continuous tick data in futures exchange market, Fractals 11 (2003) 131–136.
[24] S. Abe, N. Suzuki, Zipf–Mandelbrot law for time intervals of earthquakes, arXiv: cond-mat/0207657, 2002.