# Enhancing the Practical Usefulness of a Markowitz Optimal Portfolio by Controlling a Market Factor in Correlation between Stocks

Cheoljun Eom[1*], Taisei Kaizoji[2**], Yong H. Kim[3], and Jong Won Park[4]

1. Division of Business Administration, Pusan National University, Busan 609-735, Republic of Korea
2. Division of Social Sciences, International Christian University, Tokyo 181-8585, Japan
3. College of Business, University of Cincinnati, OH 45221, USA
4. Division of Business Administration, University of Seoul, Seoul 130-743, Republic of Korea

## Abstract

It has been suggested in previous studies that the correlation matrices created by controlling various properties included in the original correlation matrix can enhance the practical utility of the Markowitz mean-variance (MV) model. Thus, in this study we have empirically assessed whether using a controlled correlation matrix from which market factor properties in the original correlation matrix were removed could help to improve the practical utility of the MV model from two perspectives (portfolio risk reduction and diversification), according to the changes in the number of stocks in a portfolio. We found that the portfolio created from the controlled correlation matrix without the specified original market factor properties had lower risks than the portfolio from the original correlation matrix at identical given portfolio returns. We also determined that the risk reduction in the portfolio from the controlled correlation matrix was achieved via a high level of diversification of the stocks in the portfolio. The results were consistent regardless of the number of stocks in the portfolio. Accordingly, our findings showed that the use of a controlled correlation matrix with specifically removed market factor properties could actually improve the practical utility of the conventional MV model.

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[*] Electronic address: shunter@pusan.ac.kr
[**] Corresponding Author : Electronic address: tkaizoji@gmail.com

# 1. INTRODUCTION

The optimal portfolio of the conventional Markowitz (1952) mean-variance (MV) model proposed in the 1950s was impactful not only in the academic world, but also in the practical field of investment management; Markowitz’ contribution to these fields earned him the 1991 Nobel Prize in Economics. Additionally, the MV model can establish the investment weight of stocks in a portfolio, reflecting the investor's risk tolerance as well as risks and returns, thus effectively distributing the investor's wealth. This model was expected to provide portfolio managers with an effective means of distributing investment funds that could achieve anticipated portfolio returns with minimum portfolio risk. However, despite its salience to the field of finance, the MV model has been generally dismissed by portfolio managers for the past 60 years; that is to say, the model is perceived to lack practical usefulness, and is viewed by many to be of academic value only.

In some previous studies, it has been noticed that the principal reason for the cited issues with the practical applicability of the Markowitz model was the estimation error of the input data of the expected return, and the standard deviation of the stocks in the portfolio required by the MV model. Michaud (1989) claimed that the MV model is extremely sensitive to the estimation error of expected return and the standard deviation of stocks, most particularly the expected returns. As a consequence, this model can create a meaningless optimal portfolio, which if followed would result in nonsensical investment decisions. Best *et al.* (1991) also demonstrated the sensitivity of the MV model to added constraints in the optimization function, as well as to variations in the expected returns of stocks in a portfolio. Frost *et al.* (1986) pointed out that as the number of stocks in a portfolio increases, the estimation risk associated with the expected return, standard deviation, and correlation matrix among the stocks also increases. Additionally, another reason for the practical utility issues mentioned in previous studies is that the investment weights created by the MV model are not particularly well distributed among the portfolio stocks. Frost *et al.* (1988) previously provided empirical results showing that the investment weights of the MV model for a portfolio consisting of large-scale stocks (200) are distributed only to 10% or less of the stocks, regardless of investors' risk tolerance. Jorion (1992) and Chopra *et al.* (1993) also mentioned that estimation errors in the input data can cause investment weight problem in the MV model. However, interestingly enough, all of the problems identified in previous studies are problems of practical utility, as opposed to theoretical defects. Accordingly, it is necessary to exert more research effort toward a resolution of the MV model's practical usefulness problems, from both academic and practical viewpoints. The principal objective of this study, then, was to propose a method by which the practical utility of the MV model might be enhanced.

Among the studies designed to help solve the practical applicability problem, econophysicists have explored the possibility of resolving the problem with the correlation matrix among stocks—this approach is not often adopted in the field of finance. A correlation matrix measures the degree of relationship among the stocks constituting a portfolio and the measured correlation matrix exerts a direct minimizing effect on portfolio risk, which is the objective function of the MV model. However, the correlation matrix determined by a portfolio containing $N$ stocks harbours $N(N-1)/2$ elements, and it is extremely difficult to account for every element. Therefore, in the extant literature, the simplest method of estimation of the expected correlation matrix using past data has involved the assumption that the historical correlation value is a useful estimate of its expected future value. Using the random matrix theory (RMT, Metha (1991)) introduced to the finance field from the discipline of econophysics, Laloux *et al.* (1999, 2000) showed that the random components included in a correlation matrix among stocks can distort the estimation of portfolio risk, and thus controlling the random properties in a correlation matrix can result in more accurate assessments of portfolio risk. Sharifi *et al.* (2004) claimed that the risk of a portfolio from a correlation matrix whose random properties are controlled via the RMT method is stable. In other words, the applicability problem of the MV model can be overcome by controlling the properties of random components included in the correlation matrix.

The RMT method is largely identical to the multivariate statistics method of principal component analysis (PCA, Harman (1976)). Using PCA, King (1966) provided empirical evidence to suggest that the principal components reflective of the properties of a small number of large eigenvalues (including the largest eigenvalue) that explain stock price changes are market, industry, and company factors. These findings influenced the arbitrage pricing model of Roll (1976), who explained the changes in stock prices using the principle components (common factors). As another attempt at controlling the properties included in the data, Singh (1997) employed the singular value decomposition (SVD) method for the bond market to create data that reflected only the properties of a small number of large eigenvalues, and he determined that the proposed method could effectively explain bond risks using the controlled data. As previous financial studies using the PCA and SVD methods employed only the properties of the principal components, and the econophysicists using the RMT method employed other properties after controlling random components, it can be seen that the two methods are fundamentally similar. Furthermore, all of the aforementioned methods utilize eigenvalues and eigenvectors to control specific properties included in the correlation matrix.

Some recent studies have attempted verification by directly combining correlation matrices created via the control of various properties in the original correlation matrix with the MV model, in attempts to improve the practical applicability issue. Bai *et al.* (2006) previously claimed that the problem of the MV model's practical applicability is caused by the correlation matrix among stocks comprising a portfolio, as was confirmed by Laloux *et al.* (1999, 2000), rather than by estimation errors of the expected returns and of the standard deviations, as was suggested by Michaud (1989). Bai *et al.* also confirmed that the applicability of the conventional MV model can be enhanced by combining the RMT method with the bootstrap method. Eom *et al*. [2009a, 2009b] provided empirical evidence to suggest the existence of a negative relationship between the degree of portfolio diversification from the MV model and the influence power of a market factor. Based on a comparative analysis of the results of the application of the MV model to the correlation matrices created by controlling various properties included in the original correlation matrix via the RMT method, Eom *et al*. also argued that among the various properties included in a correlation matrix, market factor properties are likely to be the major causes of the MV model's applicability problem.

On the basis of the aforementioned previous studies, the objective of our study was to empirically assess whether a portfolio created via the MV model using a controlled correlation matrix without market factor properties can help to resolve the practical application problems of the conventional MV method using the original correlation matrix from two perspectives-- portfolio risk reduction and diversification-- according to the changes in the number of stocks in the portfolio. Our observed results were compared with the results of a portfolio created via the traditional approach using the original correlation matrix. A total of 23 various cases were created by changing the number of stocks in a portfolio from 10 to 400. In an effort to avoid selection bias of the portfolio stocks, simulations were conducted by selecting 100 portfolios consisting of different types of stocks, with each portfolio being comprised of an identical number of stocks. The first observation was made from the perspective of portfolio risk reduction, which involved assessing whether a portfolio with lower risk existed for given identical portfolio returns. We determined that the portfolio from the controlled correlation matrix without market factor properties evidenced significantly lower risks than were recorded for the portfolios from the original correlation matrix with the MV model. Additionally, another observation was made from the perspective of portfolio diversification perspective, which involved assessing the distribution of stocks in the portfolio. We discovered that the investment weights of the stocks of a portfolio from the original correlation matrix were biased excessively toward a limited number of stocks as the number of stocks increased, as was previously suggested by Frost *et al*. (1988). That is to say, the conventional MV model is unsuitable for determining the investment weights for a portfolio, regardless of the number of stocks in the portfolio. On the other hand, portfolios created with a controlled correlation matrix without market factor properties evidenced investment weights that were stable and well distributed among all of the stocks, regardless of the number of stocks. These results led us to conclude that the MV model's practical applicability issues might be resolved by controlling the market factor properties included in the original correlation matrix.

This paper is constructed as follows. After this introduction, Section 2 explains the data and methods used to empirically achieve the study objectives. Section 3 presents the results observed in accordance with the established empirical design. The final section summarizes the observed results and describes their implications.

## 2. DATA AND METHODS

### 2.1 Data

We utilized the daily price data of the stocks involved in the TOPIX index of the Japanese stock market. The stocks to be analyzed were selected on the basis of the following three criteria. First, the stocks had to have consecutive daily stock prices during the 18-year period from January 1990 to December 2007. Second, stocks with outliers in the descriptive statistics of |skewness|>2 and kurtosis>30 were excluded from analysis. Third, stocks in sectors with four or less companies were excluded. After the application of these three filtering processes, a total of 1,099 stocks were ultimately selected for analysis. The stock returns, $R(t)$, were calculated by the logarithmic changes of the prices, $R(t) = lnP(t) - lnP(t-1)$, in which $P(t)$ represents the stock price on day $t$.

## 2.2 Methods

This section explains the methods involved in established objectives for assessing the practical utility of a portfolio created with the MV model using a controlled correlation matrix without market factor properties, according to changes in the number of stocks in the portfolio. Observations were made from two perspectives--portfolio risk reduction and portfolio risk diversification. Thus, in order to accomplish these objectives, we constructed the empirical design in four parts as follows: first, varying the numbers of stocks in the portfolio, second, creating a controlled correlation matrix from which market factor properties are removed via the RMT method, third, conducting comparisons from the portfolio risk reduction perspective, and fourth, conducting comparisons from the portfolio diversification perspective. The empirical design implemented herein was based on the study of Eom *et al*. (2009a, 2009b).

### 2.2.1 Conventional Markowitz Mean-Variance Model

The quadratic optimization function of the conventional MV model for determining the investment weights, $w_j^p$, of stocks in order to minimize portfolio risk, $\sigma_p$, for a given portfolio return, $E(R_p)$, is presented as Eq. 1. The input data to Eq. 1 consist of expected return, $E(R_j)$, standard deviation, $\sigma_j$ and correlation matrix, $\rho_{i,j}$ among stocks in a portfolio. The outputs are the investment weights of the stocks, with which the portfolio return and risk are subsequently calculated.

$$\sigma_p = \sqrt{\sum_i \sum_j w_i^p w_j^p \sigma_{i,j}} \quad (\sigma_{i,j} = \sigma_i \cdot \sigma_j \cdot \rho_{i,j}) \tag{1}$$

Condition 1: $E(R_p) = \sum_j w_j^p E(R_j) = R_p^T \quad [\ p = 1, 2, \ldots, 50\ ]$

Condition 2: $\sum_j w_j^p \equiv 1.0$
Condition 3: $w_j^p \geq 0.0 \ \ [\ j = 1, 2, \ldots, N\ ]$

In Eq. 1, Condition 1 is the portfolio returns designed to yield target returns, $R_p^T$ from the expected returns of the stocks in a portfolio. The investment weights of the stocks that will minimize portfolio risk are determined by the changes in the target return. After establishing the range between the minimum and maximum returns that can be achieved by the stocks in the portfolio, we divide the value by 50 to acquire various target returns. In other words, the efficient frontier is created via the linkage of the combination points ($\sigma_p$, $E(R_p)$) with the minimum portfolio risks, $\sigma_p$, created for each of the 50 target returns, $R_p^T \equiv E(R_p)$. Furthermore, as our objective was to assess the practical applicability of the MV model, short-selling is not permitted under investment weight Conditions 2 and 3.

### 2.2.1 Empirical Design: Number of Stocks in a Portfolio

The first empirical design involved a method for varying the number of stocks in a portfolio. We used 23 total cases and altered the number of portfolio stocks, $N_G$ between 10 and 400, $N_G$ = 10, 20, ... , 190, 200, 250, 300, 400. Also, in order to prevent selection bias issues among the stocks in the portfolio, we conducted 100 simulations using randomly selected portfolios, such that the stocks included in the portfolios were not identical for each number of stocks. In other words, 100 portfolios, $s = 1, 2, \cdots, 100$ were selected from the combination of ${}_N C_{N_G} = \frac{N!}{N_G!(N-N_G)!}$ that could be created with $N_G$ stocks among $N = 1099$ stocks included in the TOPIX Japanese stock market. These selected stocks were then applied to the MV model of Eq. 1. This empirical approach permitted us to obtain robust empirical evidence according to the variations in the number of stocks in the portfolios.

### 2.2.2 Empirical Design: Correlation Matrix without Market Factor Properties

The second empirical design involved a method for the creation of a correlation matrix without market factor properties. Previous studies have verified that the RMT method is capable of controlling various properties included in the original correlation matrix [Laloux et al. (1999, 2000), Sharifi (2004), Eom et al. (2008, 2009a, 2009b)]. In the field of finance, controlled time series data with specific properties have been created by controlling eigenvalue via the PCA and SVD methods [King (1966), Singh (1997), Brown (1989)]. That is to say, the PCA and the SVD methods used in the field of finance, as well as the RMT method originating in econophysics and now applied to finance, are all capable of controlling the properties included in the correlation matrix. We calculated the correlation matrix, $C^M$ from which the market factor properties were removed as follows.

$$C^M = \lambda_k V_k V_k^T \quad [k = 2, 3, \cdots, N_G] \tag{2}$$

in which $\lambda_k$ and $V_k$ are the eigenvalue and the eigenvector, respectively, corresponding to the range $k = 2, 3, \cdots, N_G$. Based on the results of previous studies [King (1966), Eom et al. (2008), Brown (1989)], it has become widely accepted that among the eigenvalues created via the RMT and PCA methods, the largest eigenvalue, $\lambda_{k=1}$ has market factor properties. It has also been confirmed that the largest eigenvalue has market factor properties, regardless of any variations occurring in the number and types of stocks in a portfolio. Accordingly, as can be observed in Eq. 2, the controlled correlation matrix without market factor properties can be defined as a correlation matrix which reflects the properties of all of the eigenvalues, except for the largest eigenvalue. The controlled correlation matrix without market factor properties is used as the input data, $\rho_{i,j} \equiv C^M$ of the MV model of Eq. 1. The object used for comparison is the original correlation matrix, $\rho_{i,j} \equiv C^O$ of the conventional MV method. In this study, among the input data of the MV model, only the correlation matrix among stocks in a portfolio was differently applied, and other data regarding the expected returns and standard deviations of stocks were identical. In other words, the examination of our objective focused on the effects of the correlation matrix on the MV model.

### 2.2.4 Empirical Design: Measurements of Risk Reduction and Diversification

Next, we shall explain the empirical design of the method of comparison from the dual perspectives of portfolio risk reduction and diversification. First, the comparison for the third empirical design of portfolio risk reduction was conducted as follows. Investors prefer a portfolio which yields the minimum (maximum) risk (return) for a given return (risk). In other words, investors select a portfolio in accordance with the dominance principle. In service of the study objective, we utilized two types of correlation matrix herein—namely, the original correlation matrix and the controlled correlation matrix without market factor properties--as the input data of MV model. Other input data, including the expected return and standard deviation of portfolio stocks, were kept identical, in order to enable an objective comparison of the risks associated with the portfolios elicited from the two correlation matrices at the given identical portfolio returns, as mentioned in Condition 1 of Eq. 1. Therefore the portfolios calculated with the two correlation matrices had identical returns, and the one with the smaller risk was considered superior, consistent with the dominance principle.

The fourth empirical portfolio diversification design was as follows. Using investment weights calculated with the two types of correlation matrices, we assessed the degree of portfolio diversification. In an effort to obtain a robust evaluation of portfolio diversification, we utilized three measurements that are complementary to different interpretations: first, the intra-portfolio correlation, $IPC$, which involves the level of distribution of investment weights, second, the concentration coefficient, $CC$, which involves the level of concentration of the investment weights, and third, the comparison between numbers of stocks with and without investment weights. The three measurements can be explained as follows:

First, $IPC_p$ measures the level of distribution among the investment weights of stocks in a portfolio.

$$IPC_p = \frac{\sum_i^{N_G} \sum_j^{N_G} w_i^p w_j^p \rho_{i,j}}{\sum_i^{N_G} \sum_j^{N_G} w_i^p w_j^p} \quad [\ p = 1, 2, \ldots, 50\ ] \qquad (3)$$

$IPC_p$ is calculated using each of the investment weights elicited by the two correlation matrices $\rho \equiv C^M$ and $\rho \equiv C^O$ for $N_G$ stocks in the portfolio. With a range of $-1 \leq IPC_p \leq +1$, $IPC_p = -1$ reflects that investment weights are well distributed among all of the stocks in a portfolio, whereas $IPC_p = +1$ indicates that investment weights are not at all distributed. In other words, the smaller the $IPC_p$ value is, the higher the level of distribution of the investment weights among stocks in a portfolio will be.

Second, $CC_p$ assesses the degree of concentration of the investment weights among stocks in a portfolio.

$$CC_p = (\sum_{j=1}^{N_G} (w_j^p)^2)^{-1} \qquad (4)$$

$CC_p$ is calculated with each of the investment weights yielded by the two correlation matrices among $N_G$ stocks in the portfolio. In a range of $1 \leq CC_p \leq N_G$, $CC_p = 1$ indicates that 100% of investment is made in one specific stock in the portfolio, and $CC_p = N_G$ signifies that an equal investment weight $w_j = \frac{1}{N_G} = w$ is assessed to every stock in the portfolio. A high $CC_p$ show that weights are not concentrated on just a few stocks in a portfolio--that is, the weights are well-distributed.

Third, the numbers of stocks with and without investment weights were compared. As short-selling is not allowed in Eq. 1, in an effort to assess the practical utility of the MV model, whether or not stocks in a portfolio have investment weights is determined by $w_j^p > 0$. Among $N_G$ stocks in portfolio, the number of stocks without investment weights, $N_G^{w=0}$, is compared with the number of stocks with investment weights $N_G^{w>0} = N_G - N_G^{w=0}$. Additionally, for consistent observations according to the variation in the number of stocks in a portfolio, we utilized the ratio of stocks without investment weights, $FR_G^{w=0} = \frac{N_G^{w=0}}{N_G}$ and the ratio of stocks with investment weights, $FR_G^{w>0} = \frac{N_G^{w>0}}{N_G}$.

# 3. RESULTS

## 3.1. Comparison with Degree of Risk Reduction

This section presents the results of our comparison of the risks of two portfolios under a given return condition. One is a portfolio generated by the MV model using a controlled correlation matrix with removed market factor properties, and the other is a portfolio created via the conventional method using the original correlation matrix. In terms of portfolio selection theory, a portfolio with a lower risk for a given identical return is considered to be the superior choice. As we have designed two portfolios to yield identical returns, the results were compared from the perspective of reducing risk for the portfolios calculated with two correlation matrices. The results of these comparisons are provided in Fig. 1 and Table 1.

Fig. 1 shows our comparison of the risks of portfolios elicited with the controlled correlation matrix without market factor properties and the original correlation matrix, according to the variation in the number of stocks in the portfolio. The X-axis represents a total of 23 cases of numbers of stocks in a portfolio from $N_G = 10$ to $N_G = 400$. Y-axis indicates portfolio risks. We conducted 100 simulations, $s = 1, 2, \cdots, 100$ for 50 portfolio risks, $\sigma_{p,s}$, $p = 1, 2, \cdots, 50$ for each number of stocks in a portfolio. Fig. 1 depicts the error-bar graph drawn with the average portfolio risk

$\overline{PR} = \frac{1}{100}\sum_{s=1}^{100} PR_s$ (where $PR_s = \frac{1}{50}\sum_{p=1}^{50}\sigma_{p,s}$ ) and the standard deviation $\sigma(PR_s) = \sqrt{\sum_{s=1}^{100}[PR_s - \overline{PR}]^2/(100-1)}$. Eom *et al*. [18] verified that portfolios with low target returns evidence higher degrees of diversification than are seen in those with higher target returns. Accordingly, we separated the results of the first 25 portfolio risks, $p = 1 \sim 25$ with low target returns, $PR_s = \frac{1}{25}\sum_{p=1}^{25}\sigma_{p,s}$. In the figure, blue squares (50, $p = 1 \sim 50$) and cyan stars (25, $p = 1 \sim 25$) represent the risks of portfolios from the controlled correlation matrix without market factor properties, and red circles(50) and magenta diamonds(25) designate the risks of portfolios generated from the original correlation matrix.

The results in Fig. 1 allow us to confirm unambiguously the effects of portfolio diversification as elucidated by Evans *et al.* (1968). That is to say, the portfolio risk decreased as the number of stocks in a portfolio increased, regardless of the two correlation matrices. Comparing the magnitudes of portfolio risk reduction between the two correlation matrices under the condition of the given target returns, we determined that the risk of the portfolio from the controlled correlation matrix with the market factor properties removed was clearly less than that of the portfolio generated from the original correlation matrix. In Table 1, the risk of the portfolio from the original correlation matrix is shown to be reduced from an annual 23.68% (=$0.014979 \times \sqrt{250}$) at $N_G = 10$ to an annual 16.14% at $N_G = 400$. By way of comparison, the risk of the portfolio generated from the controlled correlation matrix without market factor properties is reduced from an annual 16.84% at $N_G = 10$ to an annual 9.42% at $N_G = 400$. Under given identical portfolio returns, the risk of the portfolio from the original correlation matrix was 1.4 (=$\frac{23.68\%}{16.84\%}$) ~ 1.7 (=$\frac{16.14\%}{9.42\%}$) times greater than that of the portfolio from the controlled correlation matrix without the market factors properties. We also determined that the risks of the first-half portfolios (25) with low target returns were smaller than those of the portfolios with all of the target returns (50). In Table 1, the risks of the portfolios from the original correlation matrix fall within a range of 19.94%~11.63%, and those of the portfolios from the controlled correlation matrix without market factor properties fall within a range of 10.53%~3.41%. Again, despite identical portfolio returns, the risks of the portfolios from the original correlation matrix were 1.9~3.4 times greater than those of the portfolios from the controlled correlation matrix without market factor properties.

These results demonstrate that a portfolio generated from the MV model using a controlled correlation matrix with market factor properties removed is superior to one created via the conventional method from the original correlation matrix, regardless of the number of stocks in a portfolio. In other words, the former is more efficient from a portfolio risk reduction perspective for given identical returns.

## 3.2 Comparison with Degree of Diversification

In order for the results observed in Section 3.1 to be considered as a possible solution to the practical applicability issues of the MV model, the portfolio from the controlled correlation matrix without market factor properties must evidence a lower risk with higher degrees of diversification of the stocks in a portfolio. Thus, using the investment weights of stocks in the portfolio shown in Fig. 1 and Table 1, we compared the degrees of diversification of the stocks in a portfolio created using the two correlation matrices. We used three indices - $IPC$, $CC$, and $w_j > 0$ - that are complementary but have different implications, in order to generate robust empirical results.

First of all, we explained the comparison of degrees of diversification between the portfolios elicited from the two correlation matrices using $IPC$ for the degree of investment weight distribution and $CC$ for the degree of concentration, as is shown in Fig. 2 and Tables 2 & 3. In the figure, the X-axis represents the numbers of stocks in a portfolio. Since we utilized the investment weight of each stock indicated in Fig. 1 and Table 1, an error-bar graph according to the average and standard deviation for $s = 100$ simulations is shown in Fig. 2. Fig. 2a depicts the error-bar graph using average $\overline{IPC} = \frac{1}{100}\sum_{s=1}^{100} IPC_s$ (where, $IPC_s = \frac{1}{50}\sum_{p=1}^{50} IPC_{p,s}$) and standard deviation $\sigma(IPC_s) = \sqrt{\sum_{s=1}^{100}[IPC_s - \overline{IPC}]^2/(100-1)}$, and Fig. 2b using average $\overline{CC} = \frac{1}{100}\sum_{s=1}^{100} CC_s$ (where, $CC_s = \frac{1}{50}\sum_{p=1}^{50} \frac{CC_{p,s}}{N_G}$) and standard deviation $\sigma(CC_s) = \sqrt{\sum_{s=1}^{100}[CC_s - \overline{CC}]^2/(100-1)}$. Although $CC$ falls within the range of $1 \le CC_{p,s} \le N_G$, we utilized the ratio against the number of stocks, $\frac{CC_{p,s}}{N_G}$ for an analysis consistent with variation in the number of portfolio stocks. Again, we separated the measurements of the first 25 portfolios with low target returns. In this figure, blue squares (50) and cyan stars (25) indicate $\overline{IPC}$ and $\overline{CC}$ from the controlled correlation matrix with the removed market factor properties, respectively, and the red circles (50) and magenta diamonds (25) represent measurements calculated using the original correlation matrix.

According to the observed results, the portfolios created from the controlled correlation matrix with removed market factor properties have higher degrees of distribution, $\overline{IPC}$ and lower degrees of concentration, $\overline{CC}$, than was observed in the portfolios generated from the original correlation matrix. Additionally, we determined that a portfolio with a low target return evidences a higher degree of distribution and a lower degree of concentration than a portfolio with a high target return. The observed results for each measurement are as follows.

First, in Fig. 2a, a smaller $\overline{IPC}$ value signifies a higher degree of distribution of investment funds among stocks in a portfolio. In this figure, we determined that the $\overline{IPC}$ calculated from the controlled correlation matrix without market factor properties was less than the $\overline{IPC}$ generated via the original correlation matrix. In Table 2, the result of the portfolio calculated from the original correlation matrix was reduced from $\overline{IPC} = 0.1848$ at $N_G = 10$ to $\overline{IPC} = 0.1442$ at $N_G = 400$. By way of comparison, the result of the portfolio created from the controlled correlation matrix with removed market factor properties changed from $\overline{IPC} = -0.0688$ at $N_G = 10$ to $\overline{IPC} = 0.0194$

at $N_G = 400$. The difference in the $\overline{IPC}$ values between the original correlation matrix and the controlled correlation matrix was 0.2536(=0.1848-(-0.0688)) at $N_G = 10$ and 0.1248(=0.1442-0.0194) at $N_G = 400$. In other words, the $\overline{IPC}$ for the stocks in the portfolio created from the original correlation matrix was 7 (=$\frac{0.1442}{0.0194}$) ~ 14 times greater than that of the stocks in the portfolio from the controlled correlation matrix with the removed market factor properties. The results showed that the degree of distribution of the conventional method was significantly lower than that of the controlled correlation matrix without market factor properties.

Second, in Fig. 2b, a greater $\overline{CC}$ value is reflective of a lower degree of concentration of investment funds among stocks in a portfolio. In this figure, we determined that the $\overline{CC}$ value from the controlled correlation matrix without market factor properties was greater than the $\overline{CC}$ value from the original correlation matrix. In Table 3, the result of the portfolio from the original correlation matrix is reduced from $\overline{CC} = 0.3804$ at $N_G = 10$ to $\overline{CC} = 0.0361$ at $N_G = 400$. By way of comparison, the result of the portfolio from the controlled correlation matrix without market factor properties is reduced from $\overline{CC} = 0.4723$ at $N_G = 10$ to $\overline{CC} = 0.1998$ at $N_G = 400$. Put another way, the $\overline{CC}$ for the stocks in the portfolio from the original correlation matrix is only 0.81 (=$\frac{0.3804}{0.4723}$) ~ 0.18 (=$\frac{0.0361}{0.1998}$) times that of the stocks in the portfolio generated from the controlled correlation matrix with the removed market factor properties. These results indicate that the degree of concentration of the conventional method is significantly higher than that of the controlled correlation matrix without market factor properties.

Next, Fig. 3 and Table 4 present the results regarding the comparison of the number of stocks with investment weights $N_G^{w>0}$ and the number of those without investment weights $N_G^{w=0}$ in portfolios generated from the two correlation matrices. As is shown in Figs. 1 and 2, Fig. 3 displays error-bar graphs using the average and standard deviations for simulation. Fig. 3a shows the results for the ratio of number of stocks without investment weights, $FR_G^{w=0}$ with an average

$\overline{FR_G^{w=0}} = \frac{1}{100}\sum_{s=1}^{100} FR_{G,s}^{w=0}$ and a standard deviation $\sigma(FR_G^{w=0}) = \sqrt{\sum_{s=1}^{100}[FR_{G,s}^{w=0} - \overline{FR_G^{w=0}}]^2/(100-1)}$. Fig. 3b shows the result for the ratio of stocks with investment weights of $FR_G^{w>0}$ using an error-bar graph with an average $\overline{FR_G^{w>0}}$ and a standard deviation $\sigma(FR_G^{w>0})$. The X-axis represents the number of stocks in a portfolio. Additionally, we separated the measurements of the first 25 portfolios with low target returns. However, as the ratio of stocks without investment weights involves only a single observation, there is no separation between the first 25 and the total 50 among the portfolios. In Figs. 3a and 3b, the blue squares (50) and cyan stars (25) designate the ratios calculated from the controlled correlation matrix without market factor properties, and the red circles (50) and magenta diamonds (25) represent the ratios calculated via the original correlation matrix.

According to the observed results, we determine that the investment weights of the stocks in a portfolio from the original correlation matrix had a problem in that they assigned weights only to an increasingly limited number of stocks as the number of stocks in a portfolio increased—this is the same problem elucidated by Frost *et al*. (1988). On the other hand, this problem did not occur in stocks within a portfolio generated from the controlled correlation matrix without market factor properties.

Fig. 3a displays the result of the ratio of stocks without investment weights among stocks in portfolios generated from the two correlation matrices. From the figure, we can observe that whereas $FR_G^{w=0}$ from the original correlation matrix increased continuously to a high value as the number of stocks in the portfolio increased, $FR_G^{w=0}$ from the controlled correlation matrix without market factor properties remained quite small, and did not significantly change. In Table 4, among stocks in a portfolio generated from the original correlation matrix, the $FR_G^{w=0}$ value increased from 9.38% at $N_G = 10$ to 84.69% at $N_G = 400$. On the other hand, the results from the controlled correlation matrix without market factor properties increased from 0.63% at $N_G = 10$ to 4.20% at $N_G = 400$, remaining at very low values without any significant variation. Comparison of the ratios of the stocks with investment weights in Fig. 3b yielded results similar to those shown in Fig. 3a. In Table 4, the result from the original correlation matrix changed from 90.63% at $N_G = 10$ to 15.31% at $N_G = 400$, while the result from the controlled correlation matrix without market factor properties changed from 99.38% at $N_G = 10$ to 95.80% at $N_G = 400$. Therefore, we determined that as the number of stocks in a portfolio increases, the portfolio from the original correlation matrix assigns investment weights to an increasingly limited number of stocks within a portfolio, but the portfolio generated from the controlled correlation matrix without market factor properties assigns investment weights to almost all of its stocks.

On the basis of the results mentioned, the comparison of the results of Fig. 3 and Table 4 with those of Fig. 2 and Tables 2 & 3 from the perspective of portfolio diversification, showed that a portfolio created from the controlled correlation matrix without market factor properties could be considered a practical method of enhancing the applicability of a portfolio generated via the conventional MV method. For example, at $N_G = 400$, the results of diversification ($IPC$ and $CC$) observed from the original correlation matrix in Fig. 2 and Tables 2 & 3 are the results of diversification among the investment weights of only 61 stocks (=$400 \times (1 - 84.69\%)$) within a portfolio. Conversely, the results from the controlled correlation matrix without market factor properties are the results of diversification among the investment weights of 383 stocks (=$400 \times (1 - 4.20\%)$) within a portfolio. Interestingly, despite the significant difference in the number of stocks with investment weights, portfolios generated from the controlled correlation matrix without market factor properties evidenced higher degrees of diversification (Fig. 2, Table 2 & 3) coupled with lower portfolio risks (Fig. 1 and Table 1). These results show that the conventional MV model is incapable of considering diversification in determining the investment weights necessary to manage portfolios containing large numbers of stocks. Otherwise, we discovered that the use of a controlled correlation matrix with the removed market factor properties is, in fact, a practical method of resolving such issues.

## 4. CONCLUSIONS

In this study, we empirically assessed whether portfolios generated from the MV model using a controlled correlation matrix with market factor properties removed could help to resolve the problems associated with the practical application of the conventional MV model identified in previous studies, regardless of the number of stocks in a portfolio, from the perspectives of portfolio risk reduction and diversification. A total of 23 various cases were created by varying the number of stocks in a portfolio from 10 to 400. In an effort to prevent selection bias of the stocks in a portfolio, we conducted 100 simulations via random selection. We utilized the daily price data for 1,099 stocks listed on the TOPIX Japanese stock market. The observed results can be summarized as follows.

First, from the perspective of portfolio risk reduction, we determined that the risk of a portfolio generated form the MV model using the controlled correlation matrix with the removed market factor properties is lower than that of a portfolio created via the conventional MV method using the original correlation matrix, at given identical portfolio returns, regardless of variations in the number of stocks in a portfolio. Consistent with the dominance principle, the use of a controlled correlation matrix without market factor properties results in a portfolio superior to one created using the conventional MV method, in terms of both risks and returns.

Second, from the perspective of portfolio risk diversification, we determined that as the number of stocks in a portfolio increased, the portfolio generated via the conventional MV method assigned investment weights to an increasingly limited number of stocks in a portfolio, whereas the portfolio generated by the controlled correlation matrix without market factor properties assigned weights to almost all of its stocks. Moreover, despite significant differences in the number of stocks with investment weights, the portfolios generated via the controlled correlation matrix without market factor properties evidenced a substantially higher degree of diversification than those from the original correlation matrix.

Our results showed that among the various properties included in the correlation matrix among stocks within a portfolio, controlling the market factor properties can effectively the practical applicability of the conventional MV model. From a practical perspective, our findings can help portfolio managers to operate large numbers of stocks with investment alternatives having high portfolio diversification and low portfolio risks, thus helping them to achieve their expected target portfolio returns. That is to say, our findings elucidate an effective means of providing a high level of practical applicability to the conventional MV model. From the academic standpoint, we believe that the results of this study provide us with a new perspective—namely, that market factor properties, which exert significant effects in many theories and models in the field of finance, can exert negative influences in case of portfolio selection. Thus, the findings of our study illustrate the need for further research concerning the correlation matrices and their values, a subject that has yet to be fully appreciated in the field of finance. Furthermore, additional research efforts will be required to extend our findings to portfolio theories and investment strategies, in order to eventually be able to make robust claims regarding the implications of this study.

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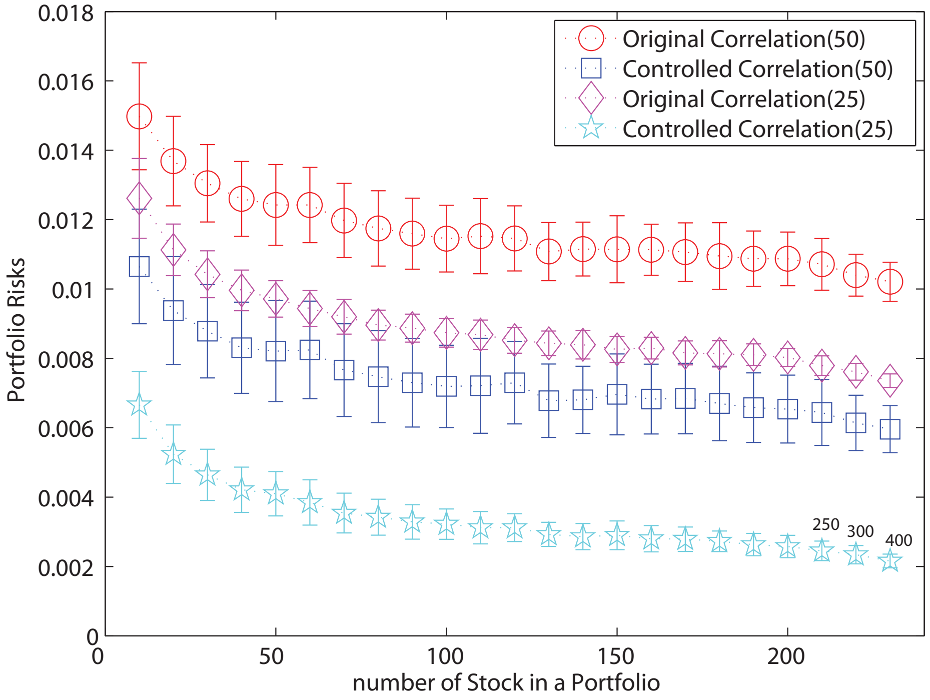

FIG. 1: (Online color). This figure shows the comparison of risks of portfolios elicited via the controlled correlation matrix without market factor properties and the original correlation matrix, according to the variation in the number of stocks in the portfolio. The X-axis represents a total of 23 cases of numbers of stocks in a portfolio and the Y-axis indicates portfolio risks. The figure depicts the error-bar graph drawn with the average portfolio risk and standard deviation. Additionally, we separated the results of the first 25 portfolio risks with low target returns. In the figure, blue squares (50, $p = 1 \sim 50$) and cyan stars (25, $p = 1 \sim 25$) designate the risks of portfolios from the controlled correlation matrix without market factors properties, and the red circles(50) and magenta diamonds(25) designate the risks of portfolios from the original correlation matrix.

| number of Stocks in a Portfolio | Total Part of Portfolios | | First Part of Portfolios | |
|---|---|---|---|---|
| | Original Correlation | Controlled Correlation | Original Correlation | Controlled Correlation |
| 10 | 0.014979$^{*}$ | 0.010650$^{*}$ | 0.012614$^{*}$ | 0.006661$^{*}$ |
| 20 | 0.013686$^{*}$ | 0.009378$^{*}$ | 0.011128$^{*}$ | 0.005237$^{*}$ |
| 30 | 0.013047$^{*}$ | 0.008784$^{*}$ | 0.010423$^{*}$ | 0.004643$^{*}$ |
| 40 | 0.012597$^{*}$ | 0.008308$^{*}$ | 0.009961$^{*}$ | 0.004212$^{*}$ |
| 50 | 0.012423$^{*}$ | 0.008210$^{*}$ | 0.009715$^{*}$ | 0.004097$^{*}$ |
| 60 | 0.012421$^{*}$ | 0.008243$^{*}$ | 0.009439$^{*}$ | 0.003846$^{*}$ |
| 70 | 0.011974$^{*}$ | 0.007661$^{*}$ | 0.009200$^{*}$ | 0.003539$^{*}$ |
| 80 | 0.011747$^{*}$ | 0.007471$^{*}$ | 0.008959$^{*}$ | 0.003421$^{*}$ |
| 90 | 0.011596$^{*}$ | 0.007296$^{*}$ | 0.008871$^{*}$ | 0.003286$^{*}$ |
| 100 | 0.011452$^{*}$ | 0.007193$^{*}$ | 0.008736$^{*}$ | 0.003222$^{*}$ |
| 110 | 0.011522$^{*}$ | 0.007211$^{*}$ | 0.008682$^{*}$ | 0.003118$^{*}$ |
| 120 | 0.011459$^{*}$ | 0.007298$^{*}$ | 0.008523$^{*}$ | 0.003120$^{*}$ |
| 130 | 0.011077$^{*}$ | 0.006781$^{*}$ | 0.008435$^{*}$ | 0.002926$^{*}$ |
| 140 | 0.011152$^{*}$ | 0.006804$^{*}$ | 0.008393$^{*}$ | 0.002861$^{*}$ |
| 150 | 0.011145$^{*}$ | 0.006962$^{*}$ | 0.008259$^{*}$ | 0.002898$^{*}$ |
| 160 | 0.011133$^{*}$ | 0.006828$^{*}$ | 0.008298$^{*}$ | 0.002801$^{*}$ |
| 170 | 0.011062$^{*}$ | 0.006843$^{*}$ | 0.008157$^{*}$ | 0.002786$^{*}$ |
| 180 | 0.010952$^{*}$ | 0.006699$^{*}$ | 0.008130$^{*}$ | 0.002727$^{*}$ |
| 190 | 0.010875$^{*}$ | 0.006579$^{*}$ | 0.008104$^{*}$ | 0.002631$^{*}$ |
| 200 | 0.010869$^{*}$ | 0.006539$^{*}$ | 0.008026$^{*}$ | 0.002575$^{*}$ |
| 250 | 0.010711$^{*}$ | 0.006440$^{*}$ | 0.007790$^{*}$ | 0.002450$^{*}$ |
| 300 | 0.010400$^{*}$ | 0.006140$^{*}$ | 0.007610$^{*}$ | 0.002341$^{*}$ |
| 400 | 0.010208$^{*}$ | 0.005957$^{*}$ | 0.007357$^{*}$ | 0.002158$^{*}$ |

TABLE 1: This table represents the results of average portfolio risks elicited using the original correlation matrix and the controlled correlation matrix without market factor properties, respectively, according to the changes of number of stocks in a portfolio from 10 to 400. Additionally, we separated the results of the total part, $p = 1 \sim 50$, and the first part, $p = 1 \sim 25$ of portfolio risks from the perspective of the magnitude of target returns. In the table, *, **, and *** indicate statistical significance at levels of 1%, 5% and 10%.

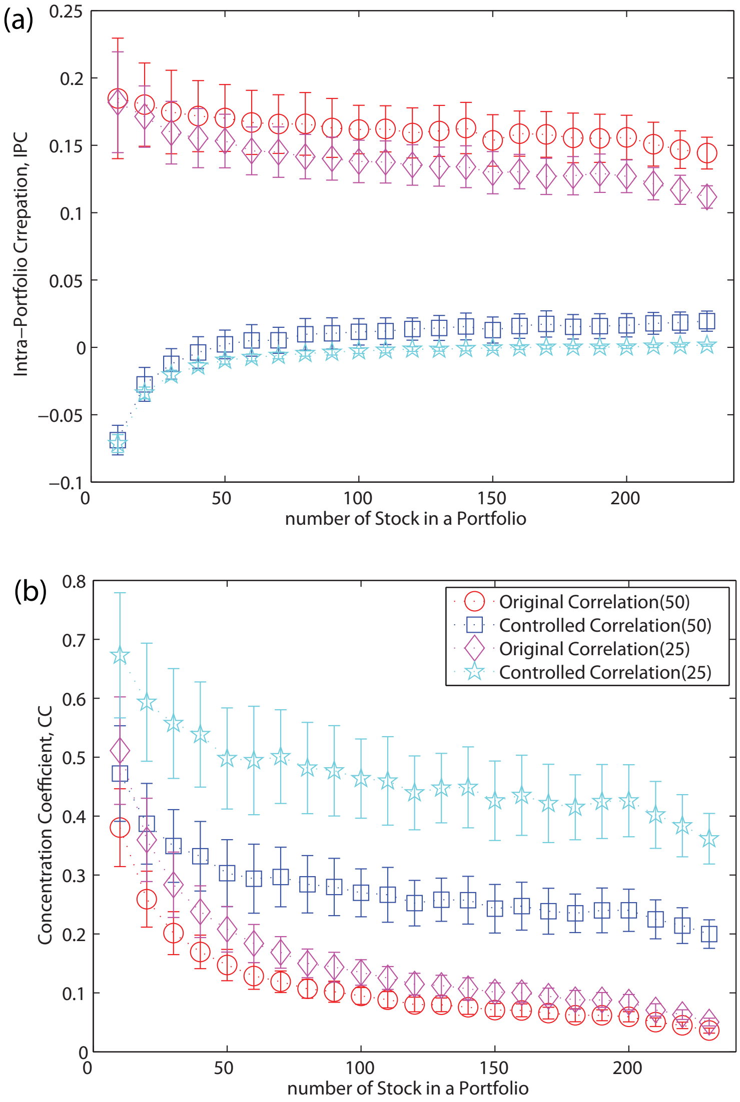

FIG. 2: (Online color). This figure shows the results of comparison of degrees of diversification between the portfolios using $\overline{IPC}$ in Fig. 2(a) and $\overline{CC}$ in Fig. 2(b). In the figure, the X-axis represents the numbers of stocks in a portfolio. The figure depicts the error-bar graph drawn with average value and standard deviation. In addition, we separated the results of the first 25 portfolio risks with low target returns. That is to say, blue squares (50) and cyan stars (25) represent the risks of portfolios from the controlled correlation matrix without market factor properties, and the red circles(50) and magenta diamonds(25) represent the risks of portfolios from the original correlation matrix in the figures.

| Number of Stocks in a Portfolio | Total 50 Portfolios | | First 25 Portfolios | |
|---|---|---|---|---|
| | Original Correlation | Controlled Correlation | Original Correlation | Controlled Correlation |
| 10 | 0.1848$^{*}$ | −0.0688$^{*}$ | 0.1820$^{*}$ | −0.0714$^{*}$ |
| 20 | 0.1802$^{*}$ | −0.0275$^{*}$ | 0.1713$^{*}$ | −0.0339$^{*}$ |
| 30 | 0.1747$^{*}$ | −0.0123$^{*}$ | 0.1594$^{*}$ | −0.0204$^{*}$ |
| 40 | 0.1717$^{*}$ | −0.0037$^{*}$ | 0.1553$^{*}$ | −0.0138$^{*}$ |
| 50 | 0.1702$^{*}$ | 0.0022$^{**}$ | 0.1532$^{*}$ | −0.0096$^{*}$ |
| 60 | 0.1670$^{*}$ | 0.0052$^{*}$ | 0.1458$^{*}$ | −0.0075$^{*}$ |
| 70 | 0.1657$^{*}$ | 0.0053$^{*}$ | 0.1450$^{*}$ | −0.0060$^{*}$ |
| 80 | 0.1659$^{*}$ | 0.0097$^{*}$ | 0.1416$^{*}$ | −0.0045$^{*}$ |
| 90 | 0.1629$^{*}$ | 0.0104$^{*}$ | 0.1401$^{*}$ | −0.0036$^{*}$ |
| 100 | 0.1617$^{*}$ | 0.0116$^{*}$ | 0.1380$^{*}$ | −0.0026$^{*}$ |
| 110 | 0.1622$^{*}$ | 0.0120$^{*}$ | 0.1376$^{*}$ | −0.0021$^{*}$ |
| 120 | 0.1592$^{*}$ | 0.0136$^{*}$ | 0.1354$^{*}$ | −0.0013$^{*}$ |
| 130 | 0.1607$^{*}$ | 0.0144$^{*}$ | 0.1342$^{*}$ | −0.0014$^{*}$ |
| 140 | 0.1628$^{*}$ | 0.0155$^{*}$ | 0.1341$^{*}$ | −0.0008$^{*}$ |
| 150 | 0.1536$^{*}$ | 0.0129$^{*}$ | 0.1296$^{*}$ | −0.0005$^{*}$ |
| 160 | 0.1587$^{*}$ | 0.0158$^{*}$ | 0.1305$^{*}$ | −0.0002 |
| 170 | 0.1581$^{*}$ | 0.0174$^{*}$ | 0.1269$^{*}$ | 0.0002 |
| 180 | 0.1555$^{*}$ | 0.0153$^{*}$ | 0.1275$^{*}$ | 0.0002 |
| 190 | 0.1552$^{*}$ | 0.0158$^{*}$ | 0.1293$^{*}$ | 0.0002$^{***}$ |
| 200 | 0.1558$^{*}$ | 0.0164$^{*}$ | 0.1270$^{*}$ | 0.0003$^{*}$ |
| 250 | 0.1508$^{*}$ | 0.0178$^{*}$ | 0.1215$^{*}$ | 0.0010$^{*}$ |
| 300 | 0.1468$^{*}$ | 0.0184$^{*}$ | 0.1170$^{*}$ | 0.0013$^{*}$ |
| 400 | 0.1442$^{*}$ | 0.0194$^{*}$ | 0.1117$^{*}$ | 0.0017$^{*}$ |

TABLE 2: This table represents the results of $\overline{IPC}$, in which a smaller value signifies a higher degree of distribution of investment funds among stocks in a portfolio, from the original correlation matrix and the controlled correlation matrix with the removed market factor properties, respectively, according to changes in the number of stocks in a portfolio from 10 to 400. Additionally, we separated the results of the total part, $p = 1 \sim 50$, and the first part, $p = 1 \sim 25$, of portfolio risks from the viewpoint of the magnitude of target returns. In the table, *, **, and *** indicate statistical significance at levels of 1%, 5% and 10%.

| Number of Stocks in a Portfolio | Total 50 Portfolios | | First 25 Portfolios | |
|---|---|---|---|---|
| | Original Correlation | Controlled Correlation | Original Correlation | Controlled Correlation |
| 10 | 0.3804* | 0.4723* | 0.5111* | 0.6729* |
| 20 | 0.2590* | 0.3871* | 0.3597* | 0.5932* |
| 30 | 0.2015* | 0.3492* | 0.2836* | 0.5572* |
| 40 | 0.1696* | 0.3319* | 0.2378* | 0.5385* |
| 50 | 0.1473* | 0.3032* | 0.2080* | 0.4979* |
| 60 | 0.1288* | 0.2938* | 0.1838* | 0.4943* |
| 70 | 0.1190* | 0.2967* | 0.1687* | 0.5010* |
| 80 | 0.1070* | 0.2844* | 0.1501* | 0.4816* |
| 90 | 0.1020* | 0.2798* | 0.1437* | 0.4769* |
| 100 | 0.0949* | 0.2699* | 0.1347* | 0.4637* |
| 110 | 0.0885* | 0.2665* | 0.1253* | 0.4597* |
| 120 | 0.0804* | 0.2523* | 0.1148* | 0.4395* |
| 130 | 0.0798* | 0.2581* | 0.1125* | 0.4475* |
| 140 | 0.0757* | 0.2573* | 0.1070* | 0.4489* |
| 150 | 0.0710* | 0.2429* | 0.1015* | 0.4260* |
| 160 | 0.0704* | 0.2472* | 0.1003* | 0.4353* |
| 170 | 0.0658* | 0.2386* | 0.0941* | 0.4215* |
| 180 | 0.0621* | 0.2351* | 0.0883* | 0.4150* |
| 190 | 0.0621* | 0.2400* | 0.0880* | 0.4248* |
| 200 | 0.0595* | 0.2402* | 0.0843* | 0.4260* |
| 250 | 0.0502* | 0.2249* | 0.0710* | 0.4021* |
| 300 | 0.0450* | 0.2143* | 0.0631* | 0.3837* |
| 400 | 0.0361* | 0.1998* | 0.0506* | 0.3616* |

TABLE 3: This table represents the results of $\overline{CC}$, in which a greater value signifies a lower degree of concentration of investment funds among stocks in a portfolio, from the original correlation matrix and the controlled correlation matrix without market factor properties, respectively, according to changes in the number of stocks in a portfolio from 10 to 400. Additionally, we separated the results of the total part, $p = 1 \sim 50$, and the first part, $p = 1 \sim 25$, of portfolio risks from the perspective of the magnitude of target returns. In the table, *, **, and *** indicate statistical significance at levels of 1%, 5% and 10%.

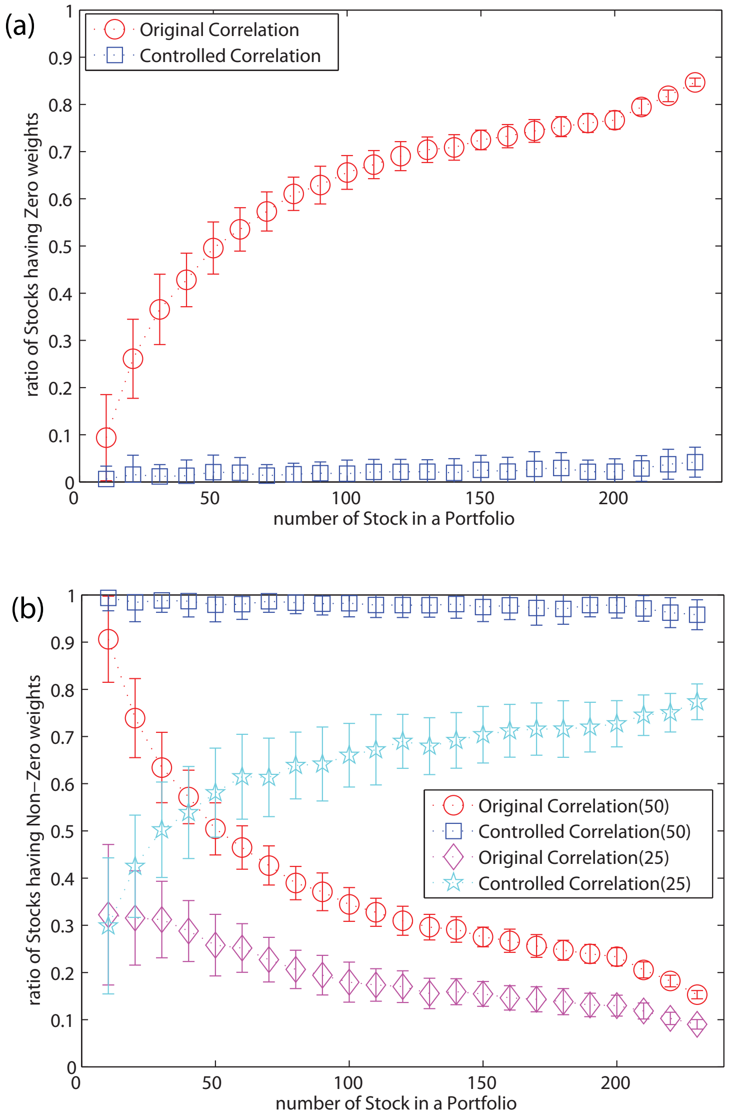

FIG. 3: (Online color). This figure represents the result regarding the ratio of the number of stocks without investment weights (Fig. 3a) and with investment weights (Fig. 3b) in portfolios. The X-axis represents the number of stocks in a portfolio. The figure depicts the error-bar graph drawn with average value and standard deviation. In Fig. 3a and Fig. 3b, blue squares (50) and cyan stars (25) indicate the ratios calculated from the controlled correlation matrix without market factor properties, and red circles (50) and magenta diamonds (25) represent the ratios calculated with the original correlation matrix.

| Number of Stocks in a Portfolio | Without Investment Weight | | With Investment Weight | | | |
|---|---|---|---|---|---|---|
| | | | Total 50 Portfolios | | First 25 Portfolios | |
| | Original Correlation | Controlled Correlation | Original Correlation | Controlled Correlation | Original Correlation | Controlled Correlation |
| 10 | 0.0938$^{*}$ | 0.0063$^{*}$ | 0.9063$^{*}$ | 0.9938$^{*}$ | 0.3225$^{*}$ | 0.2988$^{*}$ |
| 20 | 0.2611$^{*}$ | 0.0156$^{*}$ | 0.7389$^{*}$ | 0.9844$^{*}$ | 0.3156$^{*}$ | 0.4250$^{*}$ |
| 30 | 0.3657$^{*}$ | 0.0118$^{*}$ | 0.6343$^{*}$ | 0.9882$^{*}$ | 0.3121$^{*}$ | 0.5025$^{*}$ |
| 40 | 0.4282$^{*}$ | 0.0132$^{*}$ | 0.5718$^{*}$ | 0.9868$^{*}$ | 0.2879$^{*}$ | 0.5389$^{*}$ |
| 50 | 0.4956$^{*}$ | 0.0208$^{*}$ | 0.5044$^{*}$ | 0.9792$^{*}$ | 0.2579$^{*}$ | 0.5810$^{*}$ |
| 60 | 0.5352$^{*}$ | 0.0195$^{*}$ | 0.4648$^{*}$ | 0.9805$^{*}$ | 0.2519$^{*}$ | 0.6148$^{*}$ |
| 70 | 0.5732$^{*}$ | 0.0134$^{*}$ | 0.4268$^{*}$ | 0.9866$^{*}$ | 0.2272$^{*}$ | 0.6134$^{*}$ |
| 80 | 0.6105$^{*}$ | 0.0163$^{*}$ | 0.3895$^{*}$ | 0.9837$^{*}$ | 0.2064$^{*}$ | 0.6387$^{*}$ |
| 90 | 0.6291$^{*}$ | 0.0185$^{*}$ | 0.3709$^{*}$ | 0.9815$^{*}$ | 0.1943$^{*}$ | 0.6418$^{*}$ |
| 100 | 0.6558$^{*}$ | 0.0176$^{*}$ | 0.3442$^{*}$ | 0.9824$^{*}$ | 0.1795$^{*}$ | 0.6605$^{*}$ |
| 110 | 0.6724$^{*}$ | 0.0212$^{*}$ | 0.3276$^{*}$ | 0.9788$^{*}$ | 0.1736$^{*}$ | 0.6720$^{*}$ |
| 120 | 0.6904$^{*}$ | 0.0215$^{*}$ | 0.3096$^{*}$ | 0.9785$^{*}$ | 0.1700$^{*}$ | 0.6898$^{*}$ |
| 130 | 0.7039$^{*}$ | 0.0219$^{*}$ | 0.2961$^{*}$ | 0.9781$^{*}$ | 0.1556$^{*}$ | 0.6795$^{*}$ |
| 140 | 0.7088$^{*}$ | 0.0193$^{*}$ | 0.2912$^{*}$ | 0.9807$^{*}$ | 0.1593$^{*}$ | 0.6917$^{*}$ |
| 150 | 0.7247$^{*}$ | 0.0257$^{*}$ | 0.2753$^{*}$ | 0.9743$^{*}$ | 0.1546$^{*}$ | 0.7039$^{*}$ |
| 160 | 0.7327$^{*}$ | 0.0220$^{*}$ | 0.2673$^{*}$ | 0.9780$^{*}$ | 0.1463$^{*}$ | 0.7121$^{*}$ |
| 170 | 0.7437$^{*}$ | 0.0277$^{*}$ | 0.2562$^{*}$ | 0.9723$^{*}$ | 0.1434$^{*}$ | 0.7157$^{*}$ |
| 180 | 0.7529$^{*}$ | 0.0297$^{*}$ | 0.2471$^{*}$ | 0.9703$^{*}$ | 0.1382$^{*}$ | 0.7159$^{*}$ |
| 190 | 0.7605$^{*}$ | 0.0215$^{*}$ | 0.2395$^{*}$ | 0.9785$^{*}$ | 0.1312$^{*}$ | 0.7200$^{*}$ |
| 200 | 0.7668$^{*}$ | 0.0217$^{*}$ | 0.2332$^{*}$ | 0.9783$^{*}$ | 0.1298$^{*}$ | 0.7271$^{*}$ |
| 250 | 0.7946$^{*}$ | 0.0286$^{*}$ | 0.2054$^{*}$ | 0.9714$^{*}$ | 0.1184$^{*}$ | 0.7452$^{*}$ |
| 300 | 0.8181$^{*}$ | 0.0375$^{*}$ | 0.1819$^{*}$ | 0.9625$^{*}$ | 0.1027$^{*}$ | 0.7504$^{*}$ |
| 400 | 0.8469$^{*}$ | 0.0420$^{*}$ | 0.1531$^{*}$ | 0.9580$^{*}$ | 0.0902$^{*}$ | 0.7735$^{*}$ |

TABLE 4: This table shows the results of the ratio of the number of stocks without investment weights and with investment weights in portfolios generated from the original correlation matrix and the controlled correlation matrix without market factor properties, according to variations in the number of stocks in the portfolio from 10 to 400. Additionally, we separated the results of the total part, $p = 1 \sim 50$, and the first part, $p = 1 \sim 25$ of portfolio risks from the viewpoint of the magnitude of target returns. In the table, *, **, and *** indicate statistical significance at levels of 1%, 5% and 10%.