Solutions to Take-Home Quiz 2 (*September 21, 2007*)

1. What is the size of the matrix $(AB)C$.

   Sol. The matrix AB is of size $r \times t$ and C is of size $t \times u$. Hence the matrix $(AB)C$ is of size
$$r \times u.$$

2. Write the (h, k)-entry of AB.

   Sol.
$$\begin{aligned}(AB)_{h,k} &= A_{h,1}B_{1,k} + A_{h,2}B_{2,k} + \cdots + A_{h,s}B_{s,k} \\ &= a_{h,1}b_{1,k} + a_{h,2}b_{2,k} + \cdots + a_{h,s}b_{s,k} \\ &= \sum_{i=1}^{s} a_{h,i}b_{i,k} = \sum_{j=1}^{s} a_{h,j}b_{j,k}.\end{aligned}$$

3. Write the (h, m)-entry of $(AB)C$.

   Sol.
$$\begin{aligned}((AB)C)_{h,m} &= (AB)_{h,1}C_{1,m} + (AB)_{h,2}C_{2,m} + \cdots + (AB)_{h,t}C_{t,m} \\ &= \left(\sum_{i=1}^{s} a_{h,i}b_{i,1}\right) c_{1,m} + \left(\sum_{i=1}^{s} a_{h,i}b_{i,2}\right) c_{2,m} + \cdots + \left(\sum_{i=1}^{s} a_{h,i}b_{i,t}\right) c_{t,m} \\ &= \sum_{k=1}^{t}\left(\sum_{i=1}^{s} a_{h,i}b_{i,k}\right) c_{k,m} = \sum_{k=1}^{t}\sum_{i=1}^{s} a_{h,i}b_{i,k}c_{k,m}.\end{aligned}$$

   Note: From the above, we can show that $(AB)C = A(BC)$.

4. Compute the product LT. (Show work!)

   Sol.
$$\begin{aligned}LT &= \begin{bmatrix} 0 & 1 & 0 \\ 6 & 1 & 3 \\ 0 & 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 6 & -3 & 1 \\ 8 & 2 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 0\cdot 1+1\cdot 6+0\cdot 8 & 0\cdot 1+1\cdot(-3)+0\cdot 2 & 0\cdot 1+1\cdot 1+0\cdot(-2) \\ 6\cdot 1+1\cdot 6+3\cdot 8 & 6\cdot 1+1\cdot(-3)+3\cdot 2 & 6\cdot 1+1\cdot 1+3\cdot(-2) \\ 0\cdot 1+4\cdot 6+3\cdot 8 & 0\cdot 1+4\cdot(-3)+3\cdot 2 & 0\cdot 1+4\cdot 1+3\cdot(-2) \end{bmatrix} \\ &= \begin{bmatrix} 6 & -3 & 1 \\ 36 & 9 & 1 \\ 48 & -6 & -2 \end{bmatrix}\end{aligned}$$

5. Find a 3×3 matrix D such that $LT = TD$. (Solution only.)

   Sol.
$$D = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

   Note: Can you guess $AAT = A^2T$, $A^{10}T$ and A^nT?