February 29, 2000

Calculus II Final

Winter Term, AY1999-2000

ID 番号、氏名を、各解答用紙に、また、問題番号も忘れずに書いて下さい。
(Write your ID number and your name on each of your solution sheet. Do not forget to write the problem number as well.)

1. $f(x, y) = x^4 + y^4 - 2(x-y)^2$ とする。 [10pts, 15pts]

   (a) 点 $P(1, 2, f(1, 2))$ における接平面の方程式を求めよ。(Find the equation of the tangent plane at the point $P(1, 2, f(1, 2))$.)

   (b) $f(x, y)$ の極値を求めよ。(Determine relative maxima and relative minima.)

2. D を 曲線 $x = y^2$ と、直線 $x + y = 2$ で囲まれた領域とする。領域 D を図示し、次の重積分を二通りの累次積分で表せ。累次積分のいくつかの和となってもよい。(Let D be the bounded region surrounded by the curve $x = y^2$ and the line $x + y = 2$. Sketch the region D and express the following double integral by two different ways of iterated integrals. One of the expressions may include a sum of two or more iterated integrals.) [15pts]

$$\iint_D f(x, y)dxdy$$

3. 次のべき級数の収束半径 r を求め、また $x = 1$ とした級数が収束するかどうか判定せよ。(Determine the radius of convergence of the following power series, and also determine whether the series obtained by setting $x = 1$ converge or not.) [15pts $\times$ 2]

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \dots (2k-1)}{2 \cdot 5 \cdot 8 \cdots (3k-1)} x^k.$

   (b) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^{k^2} x^k.$

4. 次の積分の値を求めよ。(Evaluate the following integrals.) [20pts × 3]

(a) $\iint_D \sqrt{x}dxdy, \quad D = \{(x, y) \mid 0 \le x^2 + y^2 \le x\}.$

(b) $\int_0^1 \int_y^1 e^{x^2} dxdy$

(c) D は、四点 $(0,0), (1,-2), (3,-1), (2,1)$ を頂点とする四角形で囲まれた領域としたとき、(Let D be the region inside the quadrilateral with vertices $(0,0), (1,-2), (3,-1), (2,1)$.)

$$\iint_d \cos(2x + y)\sin(x - 2y)dydx.$$

5. 円柱 $x^2+y^2 \le ax$ $(a > 0)$ の内部にある球面 $x^2+y^2+z^2 = a^2$ の表面積。(Evaluate the surface area of a sphere $x^2+y^2+z^2 = a^2$ in the interior of the cyllinder $x^2+y^2 \le ax$ with $a > 0$. [20pts]

Suppose the surface is given by the functions $z = f(x, y) = g(r, \theta)$ on the region D, where (x, y) is for ordinary coodinate and (r, θ) for polar (or cyllindrical) coordinate. Then the surface area S is given by the following:

$$S = \iint_D \sqrt{1 + (f_x)^2 + (f_y)^2}dxdy = \iint_D \sqrt{1 + (g_r)^2 + \left(\frac{g_\theta}{r}\right)^2} rdrd\theta.$$

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