Calculus II FINAL

Winter 1995

ID 番号、氏名を、各解答用紙に、また、問題番号も忘れずに書いて下さい。
(Write your ID number and your name on each of your solution sheet. Do not forget to write the problem number as well.)

1. (x,y) が、$(0,0)$ に近づくとき、次の関数の極限は存在するか、決定し、理由も述べよ。(Determine whether the limit of the following function exists or not when (x,y) approaches to $(0,0)$. Give your reason.)

$$\frac{xy^2}{x^2+y^4}.$$

2. 次の関数の、極大値、極小値を決定せよ。(Determine maximum and minimum of the following function.)

$$xy(ax+by+c),\ (abc>0).$$

3. 次の積分の順序を変更せよ。(Change the order of the integrals of the following.)

$$\int_0^2 \left\{ \int_{\sqrt{2x-x^2}}^{\sqrt{2x}} f(x,y)dy \right\} dx.$$

4. 次の重積分を求めよ。(Find the following multiple integrals.)

   (a) $\displaystyle\iint_D (x^2+y^2)dxdy, \quad D=\{(x,y)|0\le x,\ 0\le y,\ x+y\le 1\}.$

   (b) $\displaystyle\iint_D xydxdy, \quad D=\{(x,y)|\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\}\ (a>0,\ b>0).$

   (c) $\displaystyle\iint_D \frac{y}{x+y}e^{(\frac{y}{x+y})^2}dxdy, \quad D=\{(x,y)|0\le x+y\le 2,\ x\ge 0,\ y\ge 0\}.$
   (Hint: $u=x+y,\ v=y/(x+y)$.)

   (d) $\displaystyle\iiint_V \sqrt{x^2+y^2+z^2}dxdydz, \quad V=\{(x,y,z)|x^2+y^2+z^2\le 1\}$

5. 次の級数の収束、発散を決定せよ。(Determine whether each of the following seires converges or diverges.)

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{n+1}{n^2+1}.$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1\cdot 3\cdots(2n-1)}{2\cdot 5\cdots(3n-1)}.$

6. 次のべき級数の収束半径を求めよ。(Determine the radius of convergence of the following power series.)

(a) $\displaystyle\sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n})x^n.$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}x^n.$

次の２問（７、８）の中から、１問を選択し、解答せよ。(Choose one of the following problems (7 or 8) and solve it.)

7. 次を示せ。(Show the following.)

$$\log(1+x)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}\quad(|x|<1).$$

8. 半球面 $x^2+y^2+z^2=1$ $(z\geq 0)$ の円柱 $x^2+y^2=x$ の内部の部分の面積を求めよ。(Find the area of a half sphere $x^2+y^2+z^2=1$ $(z\geq 0)$ bounded by a cylinder $x^2+y^2=x$.