February 28, 2001

Calculus II Final

Winter Term, AY2000-2001

ID 番号、氏名を、各解答用紙に、また、問題番号も忘れずに書いて下さい。
(Write your ID number and your name on each of your solution sheet. Do not forget to write the problem number as well.)

1. $f(x,y)$ を下記のものとする。(Let $f(x,y)$ be a function defined as follows.)

$$f(x,y) = \begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0). \end{cases}$$

   (a) $f(x,y)$ は、$(0,0)$ で連続かどうか判定せよ。(Determine whether the function $f(x,y)$ is continuous at $(0,0)$ or discontinuous.) [10 pts]

   (b) $f_x(0,y)$, $f_y(x,0)$ を求めよ。(Find $f_x(0,y)$, $f_y(x,0)$.) [10 pts]

   (c) $f_{xy}(0,0)$, $f_{yx}(0,0)$ を求めよ。(Find $f_{xy}(0,0)$, $f_{yx}(0,0)$.) [10 pts]

2. 次の関数の、極大値、極小値を決定せよ。(Determine relative maximum and relative minimum of the following function.) [20 pts]

$$xy(ax+by+c),\ (abc>0).$$

3. $z=f(x,y)$, $x=r\cos\theta$, $y=r\sin\theta$ とする。D を xy-平面のある領域とする（D を r,θ で表したものを D' と表すことにする）。この時、次を示せ。(Let $z=f(x,y)$, $x=r\cos\theta$, $y=r\sin\theta$. When D is a region in xy-plane, show the following. Here D' denotes the corresponding region with respect to r,θ.) [20 pts]

$$\iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dxdy = \iint_{D'} \sqrt{1+\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2}\cdot rdrd\theta.$$

4. $z = x^2 + y^2$ の、$z \leq 9$ の部分の表面積を求めよ。(Find the area of the surface $z = x^2 + y^2$ below the plane $z = 9$.) [20 pts]

5. $x^2 + y^2 = 2y$ で定義される円柱の xy-平面より上で、$z = x^2 + y^2$ より下の部分の体積を求めよ。(Find the volume of the solid under the surface $z = x^2 + y^2$ above the xy-plane, and inside the cylinder $x^2 + y^2 = 2y$.) [20 pts]

6. 次の積分の順序を変えてその値を計算せよ。(By changing the order of integration, evaluate the following.) [20 pts]

$$\int_0^4 \int_{x/2}^2 e^{y^2} dy dx.$$

7. 次のべき級数の収束半径 r を求めよ。(Determine the radius of convergence of the following power series.) [10 pts]

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)2^n} x^n$$

8. $f(x)$ を次のべき級数で定義される関数とする。すべての x について $f''(x) + f(x) = 0$ を満たすことを示せ。(Let $f(x)$ be a power series defined by the following. Show that $f(x)$ satisfies $f''(x) + f(x) = 0$ for all x.) [10 pts]

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$$

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